Exponentials and Logarithms

$y = b^x \leftrightarrow x = \log_b y \quad y = e^x \leftrightarrow x = \ln\ y$

$e = \lim(1+\frac{1}{n})^n = \sum_{n=0}^{\infty} \frac{1}{n!} = 2.71828\cdots$

$e^x = \lim(1+\frac{x}{n})^n = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

$\ln y = \int_1^y \frac{dx}{x} \qquad \ln 1 = 0 \quad \ln e = 1$

$\ln xy = \ln x + \ln y \quad \ln x^n = n \ln x$

$\log_a y = (\log_a b)(\log_b y) \quad \log_a b = 1/\log_b a$

$e^{x+y} = e^x e^y \quad b^x = e^{x \ln b} \quad e^{\ln y} = y$

Vectors and Determinants

$\mathbf{A} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$

$|\mathbf{A}|^2 = \mathbf{A}\cdot\mathbf{A} = a_1^2 + a_2^2 + a_3^2$ (length squared)

$\mathbf{A}\cdot\mathbf{B} = a_1b_1 + a_2b_2 + a_3b_3 = |\mathbf{A}||\mathbf{B}|\cos\theta$

$|\mathbf{A}\cdot\mathbf{B}| \le |\mathbf{A}||\mathbf{B}|$ (Schwarz inequality: $|\cos\theta| \le 1$)

$|\mathbf{A}+\mathbf{B}| \le |\mathbf{A}| + |\mathbf{B}|$ (triangle inequality)

$|\mathbf{A}\times\mathbf{B}| = |\mathbf{A}||\mathbf{B}||\sin\theta|$ (cross product)

$$\mathbf{A}\times\mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{matrix} \mathbf{i}(a_2b_3 - a_3b_2) \\ +\mathbf{j}(a_3b_1 - a_1b_3) \\ +\mathbf{k}(a_1b_2 - a_2b_1) \end{matrix}$$

Right hand rule $\mathbf{i}\times\mathbf{j} = \mathbf{k}$, $\mathbf{j}\times\mathbf{k} = \mathbf{i}$, $\mathbf{k}\times\mathbf{i} = \mathbf{j}$

Parallelogram area $= |a_1b_2 - a_2b_1| = |\text{Det}|$

Box volume $= |\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C})| = |\text{Determinant}|$

	SI Units	Symbols
length	meter	m
mass	kilogram	kg
time	second	s
current	ampere	A
frequency	hertz	Hz $\sim 1/$s
force	newton	N $\sim$ kg·m/s^2
pressure	pascal	Pa $\sim$ N/m^2
energy, work	joule	J $\sim$ N·m
power	watt	W $\sim$ J/s
charge	coulomb	C $\sim$ A·s
temperature	kelvin	K
Speed of light	$c = 2.9979\times10^8$ m/s	
Gravity	$G = 6.6720\times10^{-11}$ Nm^2/kg^2	

Equations and Their Solutions

$y' = cy$	y_0e^{ct}
$y' = cy + s$	$y_0e^{ct} + \frac{s}{c}(e^{ct}-1)$
$y' = cy - by^2$	$\frac{c}{b+de^{-ct}} \quad d = \frac{c-by_0}{y_0}$
$y'' = -\lambda^2 y$	$\cos\lambda t$ and $\sin\lambda t$
$my'' + dy' + ky = 0$	$e^{\lambda_1 t}$ and $e^{\lambda_2 t}$ or $te^{\lambda_1 t}$
$y_{n+1} = ay_n + s$	$a^n y_0 + s\frac{a^n-1}{a-1}$

Matrices and Inverses

$Ax =$ combination of columns $= b$

Solutions $x = A^{-1}b$ if $A^{-1}A = I$

Least squares $A^T A\overline{x} = A^T b$

$Ax = \lambda x$ (λ is an eigenvalue)

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$(AB)^{-1} = B^{-1}A^{-1}, (AB)^T = B^T A^T$

$$\begin{bmatrix} \mathbf{a} & \mathbf{b} & \mathbf{c} \end{bmatrix}^{-1} = \frac{1}{D}\begin{bmatrix} \mathbf{b}\times\mathbf{c} \\ \mathbf{c}\times\mathbf{a} \\ \mathbf{a}\times\mathbf{b} \end{bmatrix}$$

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \begin{matrix} +\,a_1b_2c_3 + a_2b_3c_1 + a_3b_1c_2 \\ -\,a_1b_3c_2 - a_2b_1c_3 - a_3b_2c_1 \end{matrix}$$

From	To	Multiply by
degrees	radians	.01745
calories	joules	4.1868
BTU	joules	1055.1
foot-pounds	joules	1.3558
feet	meters	.3048
miles	km	1.609
feet/sec	km/hr	1.0973
pounds	kg	.45359
ounces	kg	.02835
gallons	liters	3.785
horsepower	watts	745.7
Radius at Equator	$R = 6378$ km $= 3964$ miles	
Acceleration	$g = 9.8067$ m/s$^2 = 32.174$ ft/s^2	

Resource: Calculus

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