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**Exponentials and Logarithms**

$y = b^x \leftrightarrow x = \log_b y \quad y = e^x \leftrightarrow x = \ln y$   
 $e = \lim(1 + \frac{1}{n})^n = \sum_{n=0}^{\infty} \frac{1}{n!} = 2.71828 \dots$   
 $e^x = \lim(1 + \frac{x}{n})^n = \sum_{n=0}^{\infty} \frac{x^n}{n!}$   
 $\ln y = \int_1^y \frac{dx}{x} \quad \ln 1 = 0 \quad \ln e = 1$   
 $\ln xy = \ln x + \ln y \quad \ln x^n = n \ln x$   
 $\log_a y = (\log_a b)(\log_b y) \quad \log_a b = 1/\log_b a$   
 $e^{x+y} = e^x e^y \quad b^x = e^{x \ln b} \quad e^{\ln y} = y$

**Vectors and Determinants**

$\mathbf{A} = a_1 \mathbf{i} + a_2 \mathbf{j} + a_3 \mathbf{k}$   
 $|\mathbf{A}|^2 = \mathbf{A} \cdot \mathbf{A} = a_1^2 + a_2^2 + a_3^2$  (length squared)  
 $\mathbf{A} \cdot \mathbf{B} = a_1 b_1 + a_2 b_2 + a_3 b_3 = |\mathbf{A}||\mathbf{B}|\cos \theta$   
 $|\mathbf{A} \cdot \mathbf{B}| \leq |\mathbf{A}||\mathbf{B}|$  (Schwarz inequality:  $|\cos \theta| \leq 1$ )  
 $|\mathbf{A} + \mathbf{B}| \leq |\mathbf{A}| + |\mathbf{B}|$  (triangle inequality)  
 $|\mathbf{A} \times \mathbf{B}| = |\mathbf{A}||\mathbf{B}|\sin \theta$  (cross product)  

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{matrix} \mathbf{i}(a_2 b_3 - a_3 b_2) \\ +\mathbf{j}(a_3 b_1 - a_1 b_3) \\ +\mathbf{k}(a_1 b_2 - a_2 b_1) \end{matrix}$$
  
 Right hand rule  $\mathbf{i} \times \mathbf{j} = \mathbf{k}, \mathbf{j} \times \mathbf{k} = \mathbf{i}, \mathbf{k} \times \mathbf{i} = \mathbf{j}$   
 Parallelogram area =  $|a_1 b_2 - a_2 b_1| = |\text{Det}|$   
 Box volume =  $|\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})| = |\text{Determinant}|$

**Equations and Their Solutions**

$y' = cy \quad y_0 e^{ct}$   
 $y' = cy + s \quad y_0 e^{ct} + \frac{s}{c}(e^{ct} - 1)$   
 $y' = cy - by^2 \quad \frac{c}{b+de^{-ct}} \quad d = \frac{c-by_0}{y_0}$   
 $y'' = -\lambda^2 y \quad \cos \lambda t$  and  $\sin \lambda t$   
 $my'' + dy' + ky = 0 \quad e^{\lambda_1 t}$  and  $e^{\lambda_2 t}$  or  $t e^{\lambda_1 t}$   
 $y_{n+1} = ay_n + s \quad a^n y_0 + s \frac{a^n - 1}{a - 1}$

**Matrices and Inverses**

$Ax =$  combination of columns =  $b$   
 Solutions  $x = A^{-1}b$  if  $A^{-1}A = I$   
 Least squares  $A^T A \bar{x} = A^T b$   
 $Ax = \lambda x$  ( $\lambda$  is an eigenvalue)  

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$
  
 $(AB)^{-1} = B^{-1}A^{-1}, (AB)^T = B^T A^T$   

$$\begin{bmatrix} \mathbf{a} & \mathbf{b} & \mathbf{c} \end{bmatrix}^{-1} = \frac{1}{D} \begin{bmatrix} \mathbf{b} \times \mathbf{c} \\ \mathbf{c} \times \mathbf{a} \\ \mathbf{a} \times \mathbf{b} \end{bmatrix}$$
  

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \begin{matrix} + a_1 b_2 c_3 + a_2 b_3 c_1 + a_3 b_1 c_2 \\ - a_1 b_3 c_2 - a_2 b_1 c_3 - a_3 b_2 c_1 \end{matrix}$$

From	To	Multiply by
degrees	radians	.01745
calories	joules	4.1868
BTU	joules	1055.1
foot-pounds	joules	1.3558
feet	meters	.3048
miles	km	1.609
feet/sec	km/hr	1.0973
pounds	kg	.45359
ounces	kg	.02835
gallons	liters	3.785
horsepower	watts	745.7
Radius at Equator	$R = 6378 \text{ km} = 3964 \text{ miles}$	
Acceleration	$g = 9.8067 \text{ m/s}^2 = 32.174 \text{ ft/s}^2$	

**SI Units      Symbols**

length	meter	m
mass	kilogram	kg
time	second	s
current	ampere	A
frequency	hertz	Hz $\sim 1/s$
force	newton	N $\sim \text{kg}\cdot\text{m}/\text{s}^2$
pressure	pascal	Pa $\sim \text{N}/\text{m}^2$
energy, work	joule	J $\sim \text{N}\cdot\text{m}$
power	watt	W $\sim \text{J}/\text{s}$
charge	coulomb	C $\sim \text{A}\cdot\text{s}$
temperature	kelvin	K
Speed of light	$c = 2.9979 \times 10^8 \text{ m/s}$	
Gravity	$G = 6.6720 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$	



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Resource: Calculus  
Gilbert Strang

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