PROFESSOR: So now we've included all the key physics in our model-- sedimentation, deactivation of virus, filtration of the air, ventilation flows.

And now, we have to include one last piece of information, which is that all of those quantities, or many of them that we've discussed, have a dependence on the size of the droplets.

And the size of the droplets and number of the droplets is a strong function of the type of respiration which is being performed by the infected person who is spewing out or emitting these droplets, and also the respiration of the susceptible person who's breathing those droplets in.

So here is a sketch of what droplet distributions look like that have been measured for different kinds of expiratory activities.

So this is shown kind of on a log scale, at least what I'm attempting to sketch.

So the dotted line here, roughly speaking, is separating the aerosol droplets, which are less than several microns in radius and the large droplets, which are bigger than that.

And what you can see here is that as you change from resting breathing, which is let's say through the nose or even resting breathing through the mouth, this distribution starts to go up.

When you start speaking, there's a significant increase in the emissions.

Presumably because there is a fragmentation and breakup of mucus which is surrounding the vocal cords in the pharynx.

Which leads to significant aerosol emissions, which do not evaporate and do not follow the Wells curve, but actually survive.

And then that becomes much more extreme as we get towards singing and any prolonged vocalizations.

In fact, also, this size here depends on the volume of speaking.

So this is loud speaking, and somewhere below here is whispering.

So basically, the amount that you release is a strong function of the volume of speech.

And whether you're vocalizing or not makes a big difference, where vocalization leads to many more droplets being formed that are aerosols that remain in the air.

And singing is actually probably almost as bad as you can get, in terms of spewing out droplets.

What's interesting is that all these distributions have a peak or the most probable value is actually less than a micron.

So it's around maybe half a micron in diameter, which is even a quarter micron in radius.

But it can vary.

So it's somewhere in the range of, let's just say 0.3 to one micron is somewhere where you'll find the peak of these distributions.

And then they drop off, but in such a way that they're not-- they're dropping off fairly quickly in number, but since the larger droplets carry more volume, like R cubed, it turns out they're not decaying quite so quickly as a function of volume.

But still, they are decaying and the peak is in the aerosol range.

So clearly, there is a strong size dependence there which you must take into account.

But that's not it.

We've already talked about other quantities which also strong size dependence.

So we've talked about infectivity.

If we think about a water droplet, I was calculating the time for the virion to get out of the water droplet by diffusion.

And we saw that even up to 10 micron-type sizes, the virion can still escape.

On the other hand, if it's in mucus, as the droplets that are emitted from-- especially vocalization patterns-- then you'll find that, as we've already calculated, that cutoff where you start to not have time for the virus to get out might be on the order of microns.

And so we're certainly going to see that the aerosols are infectious, and that has been shown experimentally.

But we may not find that they're as infectious in the larger drop form.

So that's something we could also take into account in the model.

And then of course masks and air filters have a strong dependence on size, and that's included in their ratings.

So for example, when we go from an N95 mask to various forms of cloth face covering, there's a lot of experimental data showing the transmission of those different droplet sizes through different materials.

And in addition to the material, there's also the fit factor.

So you can have a really great material.

If you measure it, it's 99% removing the aerosol droplets.

But then you put on your face and you've got a big gap over your nose or you let it sink down below your nose.

And suddenly, you're letting a lot out as well.

So the fit of the mask is also very important, and somehow ends up being included in this quantity.

But clearly, there's a strong size dependence.

Although, I would point out, both these quantities are not varying too much over the aerosol range.

So if we're focusing on aerosols, then the variation isn't as much as you might think, actually.

So what happens with the filters is when you get to bigger than 10 microns, like 10 millimeter scale, then there's a physical blocking of those droplets and they get condensed and collected on the fibers of the material.

And so there's almost 100% of filtration at the scale of millimeters.

So when you have a large cough or sneeze, if you are wearing a mask-- even a fairly poor mask-- a lot of it gets filtered.

Yes, some drops do get through and that's been observed, but it's very significantly filtered at the high sizes.

On the other hand, when you get down to the aerosols, then the masks may not be filtering so strongly.

And the same thing with the filters.

So the HEPA filter is intended to remove the aerosol droplets, even those which are below a micron.

But the various MERV ratings of filters are not really going after the aerosols as much, but still have a significant removal.

Maybe in the order of tens of a percent, or maybe up to even 90%, depending on the rating.

So the point is, all these quantities are size dependent.

So we have to go back and revisit our model including that size dependence.

OK, so it just makes things a little bit more complicated, but not too much.

So let's go back yet again to our mass balance, but now we're going to do it in a radius resolved fashion.

And so for this, I'm in need a C of R and t, and I'm going to use the same notation, C, even though I've just changed the meaning here by including the R.

And this is going to be a radius resolved concentration of virions.

So that means it's a number of virions per air volume per radius, OK?

And so, now when we write the mass balance, instead of a dC dt with an order derivative, it's actually a partial derivative.

Because C is now a function of R and t.

So I have a partial dC dt, and then if I divide through by volume on the other side, I have a P which now depends on R.

So the production rate of droplets is certainly very strongly size-dependent, and then all these other factors may come in as well.

Divided by volume.

And then, we have a lambda C, which now depends on R times concentration.

So this is now what happens to our mass balance.

And this lambda C of R-- what does that look like?

Well, it's lambda a times one plus-- and then we have the sedimentation effect.

And I'm going to write this as R over RC squared.

Because we know the sedimentation of velocity scales like R squared.

I'll come back to this in just a moment.

This is sedimentation.

That's the tricky part.

It is strongly size-dependent.

Plus lambda v and plus Pf lambda f, which of course, depends on R as well.

Lambda v in principle could depend on R, although we haven't really considered that.

We've been thinking of more just spontaneous or maybe chemical disinfectants.

In the case of chemical disinfectants, maybe that's size-dependent too because whether the virion is attacked by a chemical depends on how big the droplet is.

So in principle, this could also be size-dependent.

So basically, everything is size-dependent.

Except for lambda a as the airflow, air changed rate.

And that, of course, does not depend on size.

Now, what have I done here in this RC?

So I've written here that if I write lambda s of R is the sedimentation rate, that's vs of R divided by the height.

So basically, that's what we talked about before as the rate at which particles are sedimenting onto the horizontal surfaces.

Where, remember H is v over a.

And then what I've done here-- notice, so what was supposed to be here is lambda s of R, but then I factored out lambda a.

So what this term is here is lambdas s of R over lambda a.

That is by definition-- so that's the same as vs of R over va, were va is the air velocity.

So that's Q, the air flow rate, the outdoor air flow rate divided by a.

And then we're writing that as R over RC squared because we know that vs scales like R squared.

So when I factor this out, I can define a quantity RC.

And what does that RC turn out to be?

That is-- basically, you can actually just see by working it through this form formula.

Actually, I'll just leave it here as a definition basically.

Now, then we can also now say what happens to these kinds of droplet distributions.

So this is kind of the important thing, is that lambda C is something which has some R dependence from filtration, as we're sketching here, but it has also very strong R dependence from sedimentation.

So we don't show it here, but the sedimentation velocity kind of increases like R squared.

So that's a pretty big dependence.

And so what that means is that if I take some of these initial droplet distributions that are coming from breathing-- so imagine this is kind of the drop of distribution soon after the aerosols leave the mouth, after the initial evaporation has taken place, but the Wells curve has been disrupted by the fact that there's mucus with lots of solute and charged molecules.

And so the droplets have reached an equilibrium distribution that looks something like is shown here.

If I then ask myself what happens at a later time, how does that build up?

So let's say one person in the room who is-- basically one of these curves is describing the distribution of droplets that they are emitting from breathing.

So let's ask ourselves what happens to the concentration profile in those cases.

So there is an RC here and R. And basically as you can see, when R is less than RC, then the sedimentation term is small.

And essentially it's just lambda a.

So that means that the removal of the infectious particles is dominated by ventilation, which is not size dependent.

So when you're less than RC, which is another way of defining the aerosol range, that is where you don't have a strong size dependence.

But when you're in the large drop category, of course those drops are sedimenting.

And in fact, this term can be very large.

If we go out to 10 microns or even up to a millimeter, the sedimentation rate is incredibly fast.

And those droplets are very quickly removed and do not end up swirling around the room, as we've been describing.

So let's imagine we take-- let's say it's somebody speaking.

And let's say that, just for illustration purposes, let's say it initially looks like this.

So this is kind of the initial profile that's in the room in early times.

Now at first, until we get to a time inverse of lambda C, that's the concentration or relaxation time.

Over that time scale, the concentration is going to be building.

But it's building the fastest in the aerosol range because those guys are not being removed.

These guys, though, are being removed.

So instead of increasing, it doesn't increase that much because they're also being removed.

So what happens at a later time is that this goes up, but not so much over there.

And it gets more and more peaked in the aerosol range because here you have basically fast removal of large drops.

But here, you have slow buildup of aerosols.

So if you wait for a timescale-- the time it takes here, time is basically lambda C inverse.

So we call that tC or tau C, that's the timescale for buildup.

That's the inverse of this.

That's how long it takes to essentially reach steady state.

And where you really get the biggest increase is among the aerosol, because over here you're basically losing them.

Now, there are filtration effects too, and there's also the infectivity.

So there are other competing size dependencies that might actually emphasize the large scale here, so I want to make sure we're clear about that.

But these size dependencies are bounded.

For example, the mass filtration is sort of bounded by one, right?

But R squared just keeps going.

So if you go to larger and larger particles, they sediment faster and faster and faster.

OK.

And so it really is true-- I didn't draw this very well, but if you get to like large sizes on the order of, let's say millimeters, those droplets really never build up in the air because they sediment almost immediately.

When you cough or sneeze, the largest droplets fall and it happens very quickly.

It might happen in a few seconds or a minute.

And we're talking about timescales here that are on the order of the air change rate, which might be on the order of hours or tens of minutes.

So basically, this size-dependent relaxation tends to kind of sharpen distribution right into the aerosol range.

Which is, again, why we're talking about indoor airborne transmission being dominated by these aerosols that are swirling around the well-mixed room.

And that's very different from transmission through coughs or sneezes of large droplets, which are only very briefly present and then they sediment out.

So if you're not standing in the way of that cough, or if the cough is blocked by a mask or a shield, then you really don't have to worry as much about that form of transmission.

But as shown here, you do have to worry about the aerosols which are building up in the air and are very strongly related to all these different factors that we've been talking about.

And in fact, maybe I should also sketch-- just to help understand this picture, maybe I should draw lambda C as a function of R.

So where this RC is-- again, I was kind of saying it in words, but it's dividing a situation when you're less than RC, where basically these quantities aren't varying too much, to the situation above where it's growing like R squared.

So basically a much faster relaxation, again, separating aerosols from large drops.

So if we substitute the Stokes velocity that we've been talking about earlier into this formula here, we can actually derive what RC is.

And it comes out to be the square root because it is defined with RC squared of nine halves, that's from the Stokes formula.

We have lambda a, the air change rate; the effective height of the room or ceiling height; the viscosity of air; and the denominator is two times the density of the liquid and gravity.

And this number it turns out is also on the order of a few microns.

So it does depend on what lambda a is, but lambda a on the order of tens of minutes to hours, this is a kind of range.

It can be smaller.

It can be bigger.

In fact, it could be-- maybe it could even be like 0.5 microns up to 5 microns.

0.5 to 5 microns is probably more accurate.

But basically, it's sitting there right about-- as I was trying to sketch here-- in the barrier between aerosols and non aerosols.

And also below RC is actually where the peak of the distributions is from typical expiratory activities.

And so the point is that those are not being affected very much by sedimentation.

Now the last thing I'll mention is that as soon as we have these kind of size-dependent properties, what happens to our calculation of transmission?

I'll just put the mathematical formula on the board here without really dwelling on it.

But the way you can plot what I've sketched down here for an actual distribution shown here would be to substitute it into the formulas we had before.

So for example, what is the time-dependent transmission rate?

Well remember, that's Qb times the integral over all the sizes now of all the size-dependent qualities.

So it's PM of R, C of R and t, and C I of R dR, where CI is the infectivity.

So basically, as you solve this [INAUDIBLE] equation here, there's sort of change of the concentration field, as I've sketched.

You have to then integrate these curves against these other factors-- the mass factor and the infectivity factor-- in order to figure out the transmission rate at that moment through the well-mixed air.

And if we solve this equation here, we can actually substitute back in and get our formula for beta, which is that beta of t is Qb squared over v.

So here I'm substituting P of R. So remember before, P was basically nd times C times the infectivity.

So that was basically the production rate of virions and we've already seen that before.

And so when I kind of substitute the solution that we've already derived, but just keeping track of the fact that these quantities are all radius-dependent and appear under the integral, we are left with the integral from zero to infinity of Pn of R squared Cv CI of R. Cv is the concentration of virions per liquid volume or mucous volume in the droplet.

CI of R is the infectivity.

And let's see, then we have also an nd of R vd of R, where vd of R is the droplet size.

All divided by lambda C of R, which is all this stuff.

And this integrand is multiplied by the exponential relaxation, one minus e to the lambda C of Rt dR.

So this is your general formula for the time-dependent transmission rate in a room where you can select an actual distribution that corresponds to the infected persons breathing activity and tells you the kinds of aerosol droplets that they are emitting, and then you basically have all these other parameters to do with filtration and airflow and ventilation, viral deactivation and sedimentation.

And you finally have to do this integral and you end up with the transmission rate.