PROFESSOR: So we've just discussed how inertia and buoyancy forces can destabilize the fluid and lead to complex flows of air in a room.

But now, let's think about how those flows will influence the transport of aerosol droplets which transmit disease.

So maybe a simpler way to think about it is imagine we have some object which is releasing a substance which could be particles containing virus.

It could also be heat or other chemical release.

And that's in a flow field similar to the types of flow fields that we've been discussing.

So that's the problem of forced or natural convection, convective transport of particles.

So here I show the problem of flow past a cylinder.

And let's say the cylinder is a hot cylinder, which is releasing heat into the fluid which I've kind of sketched by this red region here.

Now if the flow is fast compared to the diffusion of heat, you would expect a situation like this where there's a thin boundary layer of heat transfer along the front end of the sphere.

But on the trailing end, there might be a wake of kind of hot fluid that's been kind of carried away.

And that is indeed what happens when we're in the regime of dominance of convection over diffusion.

And there is a dimensionless which describes that, which is the Peclet number.

So the Peclet number is defined as the velocity times a length scale divided by D where D that is the mass diffusivity.

Now normally, D is the molecular diffusivity.

Or it's the diffusivity or let's say the droplets or whatever the individual particles are.

Now, if you want to think about diffusion of gas-- so if we have for gas molecules-- for example, like for CO2 or oxygen in the air, then the ratio of kinematic viscosity to diffusivity of the gas molecules, that's called the Schmidt number, is also around 0.7 for air.

So basically molecules of momentum are diffusing at about the same rate.

And so that tells us that for the molecules of the gas, the Peclet number is actually the same as the Reynolds number and, in fact, is very large.

If we have a larger object such as maybe a droplet, we have, of course, much lower diffusivity than the air molecules.

And we're not talking and it's a different process on a collisional.

But we're still dealing with very large Peclet numbers which will lead to kind of wakes, as I've just described here.

So it depends on the details and the size of the particles.

But for the gas, I just want to write here that Peclet number is on the same order as the Reynolds number and hence is very much larger than 1 in many cases.

Now the thing is that because the Peclet number is 1, the Peclet number is measuring the importance of convective transport to diffusion.

And so because the Peclet number is typically large, we have a dominance of convection over diffusive process.

So think of our aerosol droplets.

They do diffuse in the air.

And we can calculate that with the same Stokes-Einstein formula that we've used earlier.

But that diffusion rate is very slow compared to the sort of convective processes that are occurring in the room.

And so we typically have a high Peclet number and we see this kind of behavior.

On the other hand, we're also in the regime of high Reynolds number.

And that really changes things because then the diffusion due to sort of random fluctuations and collisions with other molecules is overwhelmed by the sort of transport and diffusion light transport that follows from vortices and turbulence.

So if I take that same cylinder and I at now at a higher Reynolds number, then we can see that the wake is not a thin little tail that extends downstream but it's really a turbulent wake where the warm fluid in this case is dispersed everywhere throughout that wake.

In fact, it's fairly uniformly mixed.

A similar situation occurs in the case of breathing, coughing, sneezing, and other forms of respiration, which we will come to shortly, where we have a relatively high Reynolds number flow, often turbulent, and we are injecting in it some respiratory aerosol particles.

And they're quite well mixed across that jet.

So we can see there's a very strong coupling between the fluid flow that we've just discussed, which is often turbulent and containing vortices and eddies, and the transport of suspended particles and droplets.

So how can we think about this problem?

So the important thing is when we get into this turbulent regime, the way an individual particle-- imagine-- I should think even here in this case.

How does an individual particle move?

It kind of follows the flow.

So it goes first through a little vortices occasionally around a big vortex, and then it does some little ones.

And so it's also doing a random walk but it's one that's driven by the turbulent flow itself.

And the length scale for the sort of steps of the random walk is actually the vortex size.

And in particular, it's dominated by the largest vortex.

So whatever the flow is, there's always a certain scale which sets the size of the largest vortex.

And so that leads to the concept of so-called eddy diffusivity in turbulent flows, which is also important in air flows.

So the eddy diffusivity is an effective diffusion-like parameter that describes the mixing and spreading of suspended particles or droplets in a flow field that is turbulent.

And in that case, we can write it two ways.

So either we have an imposed velocity u and we have length scale l, and then our eddy viscosity might be written as u times l.

So u is distance per time.

l is distance.

So it's distance squared per time.

So that has units of viscosity.

And the way to interpret this is basically sort of swirling around eddies of a size l and a characteristic velocity u.

Another way to write this would be that it's l squared over 2 times a timescale.

Because we often write diffusion, if there's a time step tau, and a length scale l for a given step, then l squared over 2 tau is the diffusivity.

That's how we think about molecular diffusivity as well.

But the question is, what is this timescale.

So we can see here that the timescale is l over u.

So it's a convective times scale.

It's the time to, essentially, go around one of those eddies.

And I'm writing these really just as scaling arguments here.

So if we look at these flows, what are the relevant length scales?

So at the beginning of this flow here, it's the length scale is that of the object.

In the case of the breathing, the length scale is initially that of the mouth opening.

But then we form these turbulent structures that expand.

And a constant we will return to you shortly is that the relevant length scale as you continue here is actually something which depends on position.

So as this thing grows the eddies are are getting bigger and bigger, and so also is faster and faster the transport by diffusion, which sort of maintains a fairly uniform concentration across that space.

And that brings us then to what happens in the whole room.

So we've been very interested in mixing in the whole room.

And so a natural picture here is to say, well, if the room has a height H, then the eddy diffusivity, which aerosol particles in the room are feeling as long as it's a fairly well mixed turbulent, more isotropic flow, could be described as the height squared over 2 times a timescale.

And the timescale should be that of the effective air change or the total air change time, basically.

So this here, again, I've written lambda bar a is the outer airflow plus the re-circulation airflow, which may be going through a filter, and divided by the total volume of the room.

So this is the total ACH.

And so what we see here then is that the formula is roughly that we should have 1/2 H squared lambda a bar.

And so this is a very simple argument based on the largest eddy is going to be, in a well mixed room, at the size of the scale of the room.

And sure enough, this relationship has actually been verified for houses and actual indoor rooms with all the furniture in them, where it's been shown that if you release a passive tracer such as carbon dioxide in the middle of the room and you have the ventilation on at a certain ACH, Air Change per Hour, lambda a bar, then this formula, even with the 1/2 actually, turns out to be a pretty good approximation for the spreading in that room as you change the size of the room and look at different rooms and also look at different air change rates.

So that's, I think, a good starting point for us as well.

At least when the room is well mixed, this is a good way to think about transport in the room.

Also, we see from this picture that the timescale for mixing is the inverse of the air change time.

So the mixing time is also comparable to the residence time of the air, including re-circulation.

So basically it's the time it takes for air to typically go through this system is also the time it takes to fully mix the system, roughly the same order of magnitude.

And that is the characteristic of a well mixed turbulent room which could occur by any of the mechanisms we've just been describing.

Although the same principles also apply to jets or strong, maybe ventilation flows past an object where you might still have some heterogeneities as I've sketched here, that we will need to consider.

The last point I'd like to come back to also is the question of sedimentation.

So you may have found it surprising that we describe the flux of sedimenting particles to the ground in a very simple way by just using the Stokes velocity, vs, and multiplying by the area.

So we said the droplet flux out of the room was just the sedimentation velocity of the droplet, which was radius dependent, times the area, the floor surface area.

And what's a bit confusing about that at first is that we know the flows are very complex in the room.

In fact, if you look at dust particles in a room as we discussed earlier in some cases, for sure, you will see them actually rising and not settling.

So they may be settling relative to the flow.

But the flow is actually convecting them upwards.

And that's here, for example, if you look at there's a dominant role in the flow that I've sketched here.

The particles over here actually might have a net velocity going up.

And over here, they're going down.

But if you decompose that velocity field, then in some cases, they're going up due to flow or convection, while they're sedimenting at the rate vs.

But then necessarily, because the fluid is approximately incompressible and is returning somewhere-- wherever it goes up, somewhere else it's coming down, we find that in other areas you have vs still pointing down with the same rate and now the flow is going down.

And if you imagine a particle that is sampling all the different velocity vectors, the blue vectors, sometimes they're up, sometimes they're down, on average the blue velocity vectors of the flow have to average to 0.

Because there's no-- or at least near 0.

There's not, let's say, a very strong vertical relative motion.

Then it's reasonable to assume that the particles will sediment out of a well mixed turbulent flow at a rate given by vsa.

And that is something that has also been validated experimentally for well mixed chambers and rooms.