PROFESSOR: So now let's focus on the steady-state transmission rate, which is really the most useful in designing a safety guideline.

It's also the most conservative because the transient transmission rate is always smaller than a steady state.

So, our formula for the steady state transmission rate is shown here, in terms of the relaxation rate, lambda_c(r), of the aerosol concentration in the air, and also n_q(r), which is the density of infection quanta in the air, per radius.

So let's sketch some of the important functions here as a function of radius and try to get a sense of how we can maybe simplify this expression.

First of all, we've already defined C_q, which is the integral of n_q(r), dr. This is the critical disease parameter, which is the infection quanta per exhaled air volume.

I don't mean to cross that out but rather just to do this.

So C_q is a very important quantity for us and we will return to that.

That's the quality that we're going to want to fit to disease data for COVID-19, specifically.

And this will be the exhaled infection quanta per air volume and we'll typically want to measure that sort of peak infectivity of an individual in order to design the conservative criteria.

So what is C_q?

If I plot this n_q, it has a bunch of factors in it.

So it has the roughly constant assumed viral load per liquid volume, it has the infectivity, which we've already argued should be smaller in larger droplets because it's more difficult for the variant to diffuse out of those droplets once you get above, say, 5, 10 microns, if not less.

There's the droplet distribution itself, which depends on the type of respiration but often has a peak which is submicron and then sort of a fairly broad tail at the higher end, with smaller amounts of larger droplets, and then V_d is 4 pi r^3, which is just the volume of a drop.

So this net quantity, n_q has some kind of peak around 1 micron or less and then a tail.

And then in the integral here, we have the integral of n_q over r is c_q, so that's very important.

But there is these other factors, p_m and lambda_c, or 1/lambda_c.

Each of those quantities gives us a cutoff which makes the larger droplets less important for this problem of airborne transmission in a well-mixed room.

So lambda_c, as you can see, is a bunch of constant factors except for the sedimentation rate.

So this lambda_a times (r/rc)^2, that is the sort of radius-dependent change of the sedimentation rate relative to the ventilation rate, lambda_a.

So as you can see, this goes like 1 plus a constant plus, r^2.

So as you go to large r, the inverse of that is 1 over a constant plus r^2, so it goes to 0.

So it provides a cutoff and the scale for that cutoff is what we called r_c, that's the critical size of a droplet, which is just sedimentary at a rate comparable to the ventilation rate because, really, this is ventilation and sedimentation which are compared when you define r_c.

In addition to that, we have (p_m)^2, which is the max penetration, or transmission factor.

So while masks are 100%, or very efficient, filtering large droplets which don't fit through the fabric or the mesh, they're not as good as filtering smaller droplets.

So if you look at the transmission probability when you're down well below micron, most masks are not doing a great job filtering, they may get 5%, 10% if you're lucky, depending the quality of the mask.

But then it comes down because you start to have better and better blockage of particles by the masks.

All these factors serve to cut off this distribution so that we're not worried about the large drops, and we're interested in aerosols.

But it does so, here, in a way which is quantitative.

So we're not just arbitrarily saying, as it's sometimes said in the field, that say 10 microns or 5 microns is the limit of the aerosols, but rather, we actually have a well-defined characteristic size that can emerge here.

And the way we can define that is by taking the full expression for the steady state transmission that is has the radius-dependent terms in it and write this as (Q_b)^2/V times-- and then we'll keep the C_q from the integral of n_r-- C_q times-- and then we'll imagine that the remaining radius-dependent factors, p_m and lambda_c, are sampled at a certain value, r-bar.

So what is r-bar?

Well if you know the function is p_m and lambda_c is a function of r, there is a value of r, which we call r-bar, which is when you actually do this full integral, you would get that value.

So that has to be determined.

It can be done numerically but you can kind of see graphically where it ends up.

What we're asking here is what is the typical value of the mask penetration factor and the relaxation time?

Well, it's going to be where the most weight is here, keeping in mind, also, that there's more volume at the higher side than at the lower side.

So if we look at how much activity there is, we might want to emphasize that.

So depending on the details here, somewhere over here is going to be r-bar.

What we're saying here is that even though our theory has all of the radius dependents in it, so if you know exactly the type of masks you have and you know p_m(r) from experimental measurements, maybe a lot about the virion and how infectious it is in different-sized droplets, or you studied sedimentation-- you have all these functions.

There's a well-defined r-bar at which you can just use this simple expression in place of actually doing those integrals.

So that's actually useful simplification.

And in addition to that, we can also write this another way, which can be useful, is to take the mask factor out and write it as a quanta emission rate, lambda_q, where lambda_q is Q_b*C_q.

This is the quanta emission rate by an infector.

So if you don't like this notion infection quanta per volume, when you multiply by the breathing flow rate, you're actually getting how many quanta per time are being emitted by the infector.

And then what's leftover is another factor, which I'll call f_d, which is something we'll come back to later, which is what I call the dilution factors.

If we take the breath of an infected individual and then it ends up being diluted into the room, the ratio of the concentration of infection quanta, or virions in the breath compared to that which emerges in the well-mixed room, that's the dilution factor.

This will become important later when we look more closely at respiratory fluid mechanics and we look at the plumes, or clouds, of droplets that are being emitted by a person when they're breathing, very close to the person's mouth it's a much higher concentration and eventually it gets sort of swirled around and mixed in the room, and it reaches the steady state values that we calculate.

This f_d gives you that ratio in some sense and gives you a sense of how bad the risk is from short-range transmission versus the well-mixed room, so we'll come back to that.

But this is a nice simplification for how we can think about the steady transmission rate in terms of several key variables, which I've boxed here.

And so we will now move on to applying this to COVID-19.