PROFESSOR: In most of our calculations of safety, we're going to be interested in the steady state average transmission rate between individuals in a room, after the aerosol particles have built up to a steady state.

But let's briefly talk about the transient buildup, and how to take that into account, just as an aside in this board here.

So here is the general expression for the transmission rate that we describe, which depends on the breathing rate squared, the volume of the room.

It's an integral over all the different drop sizes, where NQ is a lumped distribution of the number of infection quanta per volume per radius.

And then PM is the mass transmission factor, which depends on radius.

And lambda C is the total relaxation rate involving sedimentation or settling and viral deactivation and filtration.

And that also depends, of course, on R.

And that lambda C of R also ends up in the exponent here.

So that actually, that rate of relaxation of the concentration in the air, is also setting the time scale, lambda C inverse, for the buildup of those aerosol droplets in the air.

And so that's this factor here.

This is basically the transient term is this term.

And then this term, the one, is the steady term.

So we're interested now what's the effect of the transient.

Now, before we get to that, if we forget about the transient now, and we just have the steady state, then we introduce beta bar as the sort of constant steady state value transmission.

And through this definition here, by doing these integrals, we have defined an effective radius R bar, which is sort of where you evaluate the mass transmission factor, and also the filtration-- or the relaxation rate in order to make these two values equal.

So that's actually our definition of effective radius.

And so now, looking at the transient term, let's ask ourselves, what is the average transmission rate up to a certain time tau.

So that would be, we divide by a time tau, and we ask ourselves up to that time, what is the average transmission rate?

So we integrate beta dT from 0 to tau, then divide by tau.

So what would that be?

Well, we can take this time integral and bring it inside the radius integral, and write this as QB squared over B integral 0 to infinity of PM squared NQ of lambda C.

Keep in mind all those factors depend on R. Times, now, an integral from 0 to tau-- so I'll put this in brackets-- of 1 minus e to the minus lambda C, which depends on R, times T divided by divided by tau dT.

And then dR.

So switching the order of integration, where we're going to do the time integral first.

And so what we have here, if we just look only at this expression right here, we can write this as a sum of a steady state term.

So when it's just 1, this is the integral 1 over tau from 0 to tau, so that's just 1.

So that's the steady state contribution.

1 plus, and there's a transient contribution where I have to do this integral here.

So that's e to the minus lambda C of T over lambda C tau, evaluated from 0 to tau.

And so we'll come back to this in just a moment and evaluate that.

But just to draw a picture maybe first of what we're looking at here.

The average transmission rate as a function of this averaging time tau, well, eventually of course, it tends to the steady state value.

But it does so in a certain way we're going to calculate, like that, where the time for that transmission-- or for that transition, is the inverse of the relaxation time.

Although there's not a precise value of that.

But if we want to keep actually a scale for it, it's going to be evaluate at that value R bar that I mentioned.

That gives you a rough sense of the overall relaxation.

So this build up of the aerosol concentration in the room once the infected person has entered, and eventually, there's sort of a steady transmission rate to everyone else in the room.

So let's continue calculating this right here now.

So this is the transient.

And I can write this as 1.

And if I evaluate here, I can put it this way, as minus, and then I evaluate first at the lower limit, which gives me another 1, minus, and then evaluating at the upper limit, which is tau, e to the minus lambda C tau over lambda C tau.

And now, I'll use an approximation that helps me get a simple analytical results.

So I should mention, as soon as we have exponential and polynomial factors, it can be difficult to solve equations.

For example, what is the bound on the occupancy or the time in the room, or the ventilation.

We like to get a simple formula.

And so if there's a nice approximation I can make, which is that 1 minus e to the minus x over x is not too far off from 1 over 1 plus x, it turns out.

So it's not a perfect match.

You can try plotting these two functions.

But it's a reasonable approximation, given that everything we're doing in this calculation, when applied to a real situation, is going to be off by some uncertainty, which could be a factor of 2 or 3, this is actually going to be more than good enough of an approximation for us.

So if I make that approximation, then what I have here is that this thing is 1 over 1 plus x here.

And so we end up with 1 minus 1 over 1 plus lambda C tau.

And when I combine those two terms, I end up with lambda C tau over 1 plus lambda C tau.

So this is my approximation.

In fact, I can further then write that as 1 over 1 plus lambda C tau inverse, dividing the numerator and denominator by lambda C tau.

So I'm just making some approximations here that allow me to get a very simple expression in the end for my safety guideline, taking into account this transient build up here.

So remember that the bound we have is on the indoor reproductive number, which is N minus 1 times the integral to tau of beta dT.

So what is that?

That's just the sort of time average beta times tau.

So this bound is actually N minus 1, time average beta times tau.

And then our guideline, of course, is to make this less than our tolerance, epsilon.

And so what that means then is using this result, you can see that I just get the rest-- so if I look at the expression for beta bracket, it's just the steady state expression times this factor.

So basically, this is kind of the factor that corrects for transient effects.

Again, with just a simple approximation.

So I can then write that my guideline now has a modified form, which is that N minus 1 times tau is less than epsilon over the time average beta up to time tau.

And this is approximately equal to epsilon over beta steady state times this factor here.

If I multiply that to the other side, I just get 1 plus 1 over lambda C of R bar tau.

So basically, this right here is the transient correction, or modification.

And this is the steady state formula, which we will more typically be using.

Now, why do we care about the transient?

Well, first of all, you can see that by using the transient, we are being less conservative.

So if we want to be very conservative, we can say, you know what, let's just assume the second that the infected person enters the room, boom, the transition rate goes right to the maximum value.

That's the most conservative.

So generally, using the steady state is more conservative.

So that's one reason we like to use it.

Also, it gives you a simpler formula.

Why add a bunch of factors to a formula that only make it less conservative.

And we've made a lot of assumptions in this model, so it makes sense, maybe let's not worry about it.

However, I do actually like to include it for certain examples, because it allows you to capture the intuition we all have that when the time goes to 0, the risk also has to go to 0, which you don't get from a steady state.

If this bumps up right away, then you could be spending like 2 seconds in a room, and you have a chance of getting infected right away, which is actually not right.

There has to be some time physically for transmission to happen through these droplets from one person to another.

So the effect of the transient correction, as you can see here is, when lambda C tau is larger than 1-- so that's times that are kind of out here-- that term is gone.

But when you get to these earlier times, or very short times, where there hasn't been time yet for the build up of the airborne concentration, then as you see, as tau goes is 0 actually, this term diverges.

So what actually happens is that if I calculate sort of, for example, one thing you can get from this guideline is what is the sort of maximum occupancy, or versus time.

Or it's sort of the maximum time in the room for a given occupancy.

We're going to be looking at lots of plots like this.

I would have something that might look like this for steady transmission.

And when this time gets larger then lambda C of R bar inverse-- there's some critical time scale there, which is this one right here-- then when you're past that time scale, you've got the steady state.

But if you go to smaller times, then what happens at this thing can sort of blow up a lot faster.

So what it kind of helps to capture is, again, this intuition that if I put in like tau is extremely small, then of course, the risk goes away, and I can have larger numbers of people in the room, or I can tolerate smaller times and actually be safe.

So anyway, that's one reason we do that.

On the other hand, for a conservative guideline, and the most important message of this course, really, is to think about that steady state transmission rate, which we will mainly be focusing on in all of our examples.