PROFESSOR: So now let's add to our theory ways in which droplets can be removed, or infectious virions can be removed, within the room in addition to ventilation and filtration effects that we've already described.

So the primary way that can occur is simply by settling of the droplets.

So we've already talked about the Stokes settling speed, which scales as the radius of the droplets squared.

So basically, larger droplets fall fairly quickly.

And in fact, we've already discussed how they can fall to the ground in a fairly short time.

It could be on the order of minutes or less for large droplets, such as those which are spewing out of your throat when you cough or when you sneeze-- out of your nose.

But also, the smaller aerosol droplets we've already calculated can stay in the air for a very long time.

So they settle much more slowly, but they do still settle.

So we can try to see how this enters in.

And the second topic is also deactivation.

We've mentioned that the virus doesn't live forever.

So the virions in these droplets do need to find a target and get out of those droplets and into some healthy tissue to infect it within a certain period of time.

So there's a notion of a viral deactivation rate which can also be a parameter in our models.

So if we now add those two effects to our existing model-- I'll just keep rewriting our mass balance equation.

And this is the mass balance for the concentration of virions in the air.

I've also used the terminology in chemical engineering.

We're going to call this kind of approximation the CSTR, or the continuously stirred-tank reactor approximation.

And now with all the effects we're including, it's starting to look more like actual modeling of chemical reactors and chemical plants by this method.

So the mass balance tells me that the volume of the room times dc/dt-- or again, c is the virion concentration per volume in the air-- is the production rate, P, minus-- and then we have a flux, which is the flow rate times the concentration.

And there are several flow rates here.

There's Q, which is ventilation.

There is filtration, which is PF QF.

And then we have now a new term, which I'll write in another color-- plus vsA.

So that's the settling here.

So that idea is, in a well-mixed room, there is a complex flow profile which is leading to the mixing by convection of the air.

And you might say, well, OK.

That flow is very quickly carrying the particles up, carrying them down-- but on average, the particles go down just as much as they go up.

And if it's well-mixed, then the particles essentially are sampling the whole space.

And relative to that well-mixed flow, which averages to 0, they are slowly settling.

And so a reasonable approximation is to say, well, the removal is basically happening with a flux rate, which is that velocity of falling times the area.

That's how quickly those particles are falling through any horizontal surfaces make relative to their average 0 motion from convective mixing.

So this is the new effect of sedimentation.

And then actually, I will also add to that another new term, which is the deactivation.

And so here, we will also add lambda v times the volume.

So this is just saying that throughout the whole volume of the room, there is a rate at which every virion is just slowly deactivating.

That will be lambda v. Also, if you have any volumetric treatments of the air, such as chemical disinfectants or even UV light, that may also slowly deactivate the virus or the virions in the air with a term that goes like this.

So I'll just mention that maybe briefly here.

So lambda v is the virion deactivation rate.

And well, if we look at tv which we've talked about before, which is lambda v inverse, this is the deactivation time.

This thing has been measured to be of order of 1 hour in some studies.

But also , even greater than 16 hours in aerosol form in other studies for SARS-CoV-2.

So it could be potentially long.

Also, this could include effects such as I mentioned-- UV light treatments, which might be operating in a certain part of the room, but then the air circulates and we're essentially treating a significant part of the volume.

It could also be chemical disinfectants.

So there are various chemicals that can be sprayed in the air which are believed to essentially kill the virus or deactivate the virions, although they may cause other harmful effects, and so it's not so widely used.

But in principle, that would also appear in our simple model, lumped into lambda v.

So let's put all these effects together now.

So again, we haven't really changed the calculation much.

We're just building it up and making it a little more complicated each time.

So let's see here.

So one thing we did with this equation is we divided both sides by v. And so let me write this equation again after such a division.

That would be dc/dt is equal to-- well there's P/v. But then we have over here-- Q/v is our lambda.

But notice, all these things are essentially giving us a correction to lambda, the relaxation time.

And actually, I should say this is a minus sign.

So we get minus lambda.

And I'll say lambda c, just for the relaxation rate of the concentration field.

So we can lump all these parameters and we can write lambda c is-- Well, from the first one, Q/v is lambda a.

That's the air change rate of outdoor fresh air.

There is PF lambda F, which is the rate of filtration times the filtration efficiency, PF.

And then we have another term which we can write as-- well, we have lambda v. That's an easy one.

And then the sedimentation term is the one I want to focus on right now.

That is vs. We can write it as vs/h, where I've divided by v and I'm writing v/a is h.

So I'm writing h equals v/a.

So if we have a rectangular box of a room, then h is the ceiling height.

But this is some kind of effective ceiling height if it's not a perfect box shape.

But if you take the volume and divide by the projected area of horizontal surfaces, then that's giving you a sense of the typical height.

And that's the typical distance by which particles have to fall.

And notice, velocity is distance per time.

So when I do vs, and divide by h I am getting something with units of inverse time.

So it's just like all the other lambdas.

It is basically a rate-- something per time.

So this is the concentration relaxation rate.

I guess it would be the theory on concentration in the air, which is relaxing at this rate, lambda c.

And then we come back to solving the same simple order of differential equation that we've done all along.

And the solution's just c of t is a steady state value times 1 minus e to the minus lambda ct, assuming that this thing is a constant for the moment.

And also, we know that the cs is p over v lambda c.

So basically, if lambda c is high, if all these removal rates are high, then that makes cs low.

So the background concentration of the room is much smaller if these lambda rates are all high.

Also, if the lambda rates are high, then the relaxation is very fast, so you very quickly get to the final value.

And so that's actually worth sketching what that looks like.

So if I plot what is the concentration, c, as a function of time, we have here this lambda c inverse is the overall concentration relaxation time.

So it looks like an exponential relaxation to a value cs.

But I just want to emphasize what I just said verbally, looking at these equations, is that as I vary lambda c-- so if I have a fast relaxation.

Let's say lambda c is a large value.

Then I start out at the same rate, but I saturate a lot faster and at a lower value.

So if it's here, this is fast lambda c.

So now if I increase lambda c relative to the blue curve, the whole thing comes down.

But also, it relaxes more quickly.

On the other hand, if I have slow relaxation, because any of these processes here are slow, then I get something which relaxes much more slowly and ends up at a higher value.

So if someone is breathing infectious air out, exhaling, the infected person, and there's only very slow processes in the room which are removing that infectious air, then it's slow to build up, but it keeps building and building and building until it finally saturates.

So this is basically, whenever lambda c, is not a large value or slower, you have a slow process, but it also builds up a lot higher.

So this effect of lambda_c is very important to keep in mind, especially because these parameters here are not necessarily constants.

In particular, v_S has a very strong dependence on size.

So these different kind of saturation curves are, at the very least, dependent on the size of the droplets that we're talking about.

And there's not just one size, so we will come to that point.