

MITOCW | MITRES_10_S95F20_0412_300k

PROFESSOR: So we've just discussed the strategies for reopening schools and businesses based on the indoor safety guideline for the given space and the various physical parameters in that space.

And the new concept we added was a prevalence of infection.

So we would know, on average, how many infected people we might expect in a room.

And as that number goes to zero, as the pandemic subsides, we can switch from a more restricted situation with an occupancy N_1 , prescribed by the original guideline based on the indoor reproductive number, to kind of increasing occupancy up to N_0 , which is the normal original occupancy, initially with masks.

And as the prevalence goes down, we switch to taking the masks off and really returning back to normal.

So I'd like to take this a little further now and ask, how would we change our policies or this discussion here if we not only have information about the prevalence of infection but we also have an understanding of immunity, which could be acquired through vaccination or through previous exposure?

And especially right now as I'm recording this lecture in early 2021 and several vaccines have rolled out for COVID-19, this is a topic of great interest.

So let's think about, how would we take into account susceptibility?

So, in some sense, this was a conservative estimate of the true risk of transmission because we've assumed that everyone who's uninfected is susceptible.

But, of course, as immunity is increased in the population, we're going to have to modify that.

So instead of having our populations of susceptible and infected persons being sampled from a two-state or two-category process, we can think of three categories.

There can be-- P_I is the probability that a person is infected, which means really, again, that they're infectious and they can affect other people.

And this is coming from the local population that is entering that indoor space.

And now we're going to add P_S , which is the probability that a person is susceptible.

And then the third category is P_M , which is the probability that the person is immune.

And so we have a three-category process, so those three should add up to 1.

So this is $1 - P_I + P_S$.

And we can also further write this as P_{VAC} , the probability that a person has been successfully vaccinated and actually has acquired immunity, plus the probability of previous exposure.

We'll call that P_X .

So this would be vaccination, and this would be previous exposure if that previous exposure has actually led to immunity.

And that's a controversial topic, still, under research and may depend on the specific population at hand.

But let's imagine that we have subsets of what these numbers are and then we'd like to see how to adjust our thinking here.

So we're still going to base our guideline on saying that the expected number of transmissions is the expected number of infected time susceptible, so the expected number of pairs, times the average transmission rate, average of beta times tau, the time, and that that expected number of transmissions should be less than our tolerance epsilon.

So that's still our guideline.

So what we're really trying to consider, now, are different assumptions about this expected number of infected susceptible pairs that are in the room.

And we've broken that down into three risk scenarios.

And let's revisit that, now, with our three-category model.

So the first risk scenario was describing a desire to limit spreading of the disease through this indoor space.

This is our original goal.

And by that, we mean, if an infected person enters the room, then we would like to make sure that it's unlikely that a new case would emerge from transmission from that person.

So in that case, we have I is equal to 1.

And then, now, I is known.

So this is just the expected value of S .

And the expected value of S , though, in this new model, is the number of other people in the room, n minus 1, times PS .

So you see, now, when I define my indoor reproductive number as N minus 1 times beta tau and I want to bound that to be less than epsilon-- that's my typical guideline-- there's this extra factor, PS , which could be moved to the other side.

So one way to think about it is, since PS is less than 1, we are increasing that tolerance because there are fewer susceptible people.

So we're allowed to stay in the room longer, have a higher occupancy, lower ventilation, et cetera.

So this is one case.

The next case is to limit transmission.

So here we're not going to assume that an infected person actually is there, but we are going to consider the possibility that there is an infected person there.

So that makes transmission potentially a lot less likely.

And so what we'd like to do here is to look at the expected value of I times S .

And I won't go through the details.

But for the trinomial distribution with three independent possibilities, with these probabilities-- and you're making N samples from that distribution-- you can show that the expected value of this product is actually N minus 1 times P_I times P_S , by very similar arguments as we have done for the binomial case.

One way to think about this is that N minus 1 is the number of permutations of two people that can be made in that room.

So if I pick one person to be first the infected and the other one to be the susceptible, this is the number of such pairs.

And $P_I P_S$ is the probability of each of those instances.

So this is the expected number of I to S pairs.

And I put a directionality here because we are distinguishing each individual person.

So if I take two people, I am counting differently.

If one is infected, the other one's susceptible or the reverse situation since everyone's a unique individual.

OK, so if we then substitute into this formula, then, notice, now, we've picked up some extra factors.

So now the guideline would read that R_N will be less than ϵ over N times P_I , which is something we already had before.

But now there's also a P_S .

So that's modified.

And then, finally, our third risk scenario was to limit personal risk.

So this is the case where S is equal to 1.

I'm only worried about one susceptible person, and that's me.

And I'm then-- if S is known, then we just have the expected value of I , which is just N minus 1, all the other people, times P_I .

And if you plug that into the formula, then you find that R_N is now bounded by ϵ divided by P_I .

So take into account both prevalence of infection and susceptibility, at least to these somewhat modified bounds.

And let's focus on where the changes took place.

So first of all, in the original guideline for limiting spreading, we can be a little bit more lenient.

So as there's more vaccination and more immunity, we don't need to keep holding that guideline at the same level, even in the most sort of conservative stance of trying to limit spreading.

What's more interesting for this plot here is the middle one.

So now, we again think up a factor of P_S .

But everything else is the same.

So what it means is that, relative to the calculation that I showed here, I should actually make the very same plot where I don't just plot PI in this axis, but actually plot PI times PS , where PS is the probability of being susceptible, which is related to vaccination and previous exposure rates.

So that actually does bring this down and, hence, make it easier to make the decision to relax restrictions and even ultimately take off the mask, because as a combination of these two factors, we're getting even more safe.

Interestingly, down here for personal risk, we don't really care about the probability of susceptibles because the only person I care about in that situation is myself.

And so I don't have any effect of susceptibility, only the effect of infection.