

## Case Study

### Using a Probabilistic Model to Investigate Financial Risks

*This example shows how to create a simple spreadsheet that can be used to explore risk and uncertainty using random variables for the key factors in the analysis.*

When evaluating a potential project, owners and investors will naturally be concerned about making enough profit to achieve their minimum acceptable rate of return. If they are borrowing money for the project, then they want the profit from operations to be large enough to cover interest payments on the debt that they have incurred while also providing an acceptable rate of return on their equity, i.e. what they themselves have invested.

(Eq. 1)            Investment = Equity + Debt

(Eq. 2)            Profit = Revenue – Cost – (Interest rate)(Debt)

(Eq. 3)            ROI = Profit/Equity

In any project, there will, at the outset, be considerable uncertainty related to each of the variables in these equations. The total investment will not be known until construction is completed, and the amount of debt and the interest rate on the debt will not be known until financing is arranged. Actual revenues and operating costs will be uncertain for some time after project completion, as it may take some years for the entire project to reach full operation.

It is possible to treat all of the variables in Equations 1 to 3 as random variables. Each time that a random variable is encountered in an analysis, a value for that variable is generated from a set of possible discrete values or a possible distribution of values. If there are more than one random variables in a set of equations, then they could be independent or dependent. If they are independent, then the value selected for one variable does not affect the value selected for the other variable. If they are dependent, then the value selected for one variable will influence the value selected for the other variable.

Random variables can be created in spreadsheets by using the function that generates random numbers. In Excel, for example, it is possible to use the function RAND() to generate a random number greater than or equal to 0 and less than 1.0. Whenever the spreadsheet is recalculated, a different random number will be returned. This function can be used to create random variables that take on a value that is believed to be uniformly distributed between a minimum value A and a maximum value B:

(Eq. 4)            Random Value between A and B = A + RAND() (B-A)

A random variable can also be defined to take on discrete values. For example, suppose that a random variable could take one of two equally likely values  $V_1$  or  $V_2$ . This random variable can be created in a spread sheet as follows. First select a random number by using RAND(), then create a scheme such as this:

(Eq. 5)            Discrete Random Variable =  $V_1$  if  $0 \leq \text{RAND}() < 0.5$

(Eq. 6)            Discrete Random Variable =  $V_2$  if  $0.5 \leq \text{RAND}() < 1$

If the values are not equally likely, then it is possible to adjust the ranges in these equations. For instance, if  $V_1$  is three times as likely as  $V_2$ , then:

(Eq. 7)            Discrete Random Variable =  $V_1$  if  $0 \leq \text{RAND}() < 0.75$

(Eq. 8)            Discrete Random Variable =  $V_2$  if  $0.75 \leq \text{RAND}() < 1$

This process can readily be extended to allow any number of discrete values, each with their own probability, by dividing the interval between 0 and 1 into increments such that the length of each increment equals the desired probability for each value. Thus, we can now create a random variable R that takes of values  $V_1, V_2, \dots V_n$  with probabilities  $P_1, P_2, \dots P_n$ .

It is possible to create new random variables as the sum, ratio, or other functions of multiple random independent variables. Thus, if investment, revenue and cost are independent random variables, then profit and ROI will also be random variables. It is also easy to create a random variable that is dependent upon the value of another random variable. This can be done by using statements such as this in a spreadsheet:

(Eq. 9)  $R_2 = a + b R_1$

In this case, the value of  $R_2$  will depend upon the value of  $R_1$ . More complex equations can also be used to allow different calculations for  $R_2$  depending upon the value of  $R_1$ . For example:

(Eq. 10) If  $R_1 < A$  then  $R_2 = 100 + R_1$  but if  $R_1 \geq A$ , then  $R_2 = 50 + R_1$

With some ingenuity, it is possible to design quite complex simulations using a spreadsheet. A time-based simulation can be structured by having values of variable in one time period depend upon the situation at the end of the previous time period. If all the variables, random numbers, and calculations are included in one (possibly very long) row of the spreadsheet, then copying that row over a range of rows will produce a simulation of how the system will change over time. The following hypothetical example demonstrates how random variables can be used to estimate the probability that a proposed project will be acceptable.

Suppose a company is considering making an investment of \$100 million to purchase the rights to operate an existing toll road. Toll revenues are expected to range from \$20 to \$25 million annually, while operating and maintenance expenses are expected to range from \$10 to \$15 million annually. The company plans to sell bonds to raise \$80 million at 8% interest and raise \$20 million from equity investors who anticipate a 10% ROI. Assuming that revenue and operating costs are independent random variables, what is the likelihood that the ROI will exceed 10% in any given year? What is the likelihood that net revenues in any year will be insufficient to cover the interest costs? A quick glance at the minimum expected revenue and the maximum expected cost shows that the annual revenue could be as low \$5 million, which would be insufficient to cover the \$6.4 million interest on the bonds. Is this something that investors should worry about?

To answer this question, revenue and cost can be defined as independent random variables. Investment cost, total debt, and interest rate on debt are assumed to be \$100 million, \$80 million and 8%. Profit will be as follows:

(Eq. 11) Profit = Revenue – Cost – 8% (\$80 million)

Table 1 shows results from a probabilistic model that was created in a spreadsheet. The table has 20 rows, each of which could be viewed as a separate year over a 20 year period or 20 different random results for a single year. Investment, debt and interest rates are assumed to be as given, but the spreadsheet has an area at the upper right where these factors can be modified. Each row of the main body of the spreadsheet has two random variables. The first is used to calculate a value for revenue and the second is used to calculate a variable for cost, both using Equation 4. Profit is calculated using Equation 11 and ROI is calculated as profit/equity (Equation 3). The average ROI is calculated as the average profit divided by the net investment of \$20 million. According to this analysis, the financial aspects of the project are solid. There is very little chance that net revenues will be unable to cover costs, and the expected ROI is over 20%, well above what the investors have been promised. The annual values vary considerably, reflecting the uncertainty in the estimates of revenue and cost. However, the risk of failure is very low.

Once this spreadsheet was created, it was possible to conduct further analyses within a few minutes. For example:

- Running the analysis 10 times produced average ROI ranging from 16.8% to 23.2%; the median ROI was 18.1%.
- If interest rates were increased to 9%, the average ROI ranged from 8.7% to 18.4%; the median ROI in 11 trials was 14%. Increasing interest rates on the bonds naturally reduces ROI, but the project still can be expected to have better than a 10% return, which is what the investors are looking for.
- If there is a cost overrun, and the investors have to put up an additional \$20 million, the ROI ranged from 6% to 11.4% with a median of 8.6%. Thus, controlling construction costs appears to be quite important.

**Table 1 A Spreadsheet That Uses Random Numbers to Explore Risks and Uncertainties Associated with a Project**

		Investment	\$100 million				
		Debt	\$80 million				
		Interest	8%				
Year	Random Number 1	Random Number 2	Revenue	Cost	Interest	Profit	ROI
1	0.887713	0.138114	24.44	10.69	6.40	7.35	36.7%
2	0.997132	0.077517	24.99	10.39	6.40	8.20	41.0%
3	0.651748	0.191994	23.26	10.96	6.40	5.90	29.5%
4	0.474722	0.464287	22.37	12.32	6.40	3.65	18.3%
5	0.922596	0.886441	24.61	14.43	6.40	3.78	18.9%
6	0.415191	0.818556	22.08	14.09	6.40	1.58	7.9%
7	0.739819	0.764117	23.70	13.82	6.40	3.48	17.4%
8	0.425825	0.106793	22.13	10.53	6.40	5.20	26.0%
9	0.771215	0.067184	23.86	10.34	6.40	7.12	35.6%
10	0.590406	0.437435	22.95	12.19	6.40	4.36	21.8%
11	0.607368	0.91192	23.04	14.56	6.40	2.08	10.4%
12	0.005216	0.406998	20.03	12.03	6.40	1.59	8.0%
13	0.382789	0.919088	21.91	14.60	6.40	0.92	4.6%
14	0.003209	0.711268	20.02	13.56	6.40	0.06	0.3%
15	0.479641	0.470464	22.40	12.35	6.40	3.65	18.2%
16	0.172015	0.24961	20.86	11.25	6.40	3.21	16.1%
17	0.648983	0.107807	23.24	10.54	6.40	6.31	31.5%
18	0.065481	0.587854	20.33	12.94	6.40	0.99	4.9%
19	0.930513	0.395585	24.65	11.98	6.40	6.27	31.4%
20	0.94164	0.431054	24.71	12.16	6.40	6.15	30.8%
Total			455.57	245.72	128.00	81.85	20.5%

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Carl D. Martland

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