Class 13
Number: A "Special" Domain of Mind & Brain?

O. A few comments on the Midterm

I. Understanding "Approximate Number System" (ANS)

- Adults
- Infants
- Animals

II. Brain basis of approx number:

Specific regions for number/magnitude?

- neuropsychological patients
- fMRI understanding approximate number
- TMS
- "Number neurons"

The Concepts of Number and Quantity

We all use concepts of number and quantity every day:
to calculate the change given to us in the store,
to tell time,
to choose the largest cookie or
the shortest line in the grocery store.

Elaborations of the concepts of number and quantity
have given rise to modern industrialized societies,
featuring
engineering
modern science
computer science

Animals too are capable of mastering simple concepts
of order, number, and quantity.
they need to, for lots of reasons…

Concept of Number/Quantity in the Wild

Foraging, maximize:

amount, probability & rate

of food.

Forming teams

- Schooling fish can quickly pick the more numerous group to join, statistically reducing their chance of getting eaten.
- African lions, spotted hyenas and wolves can assess the number of conspecifics calling and respond based on numerical advantage.

Mating: The male Tungara ("n + 1") frog matches or one-ups the number of "chucks" in neighbor's call to impress females

How is this computed in mind & brain?

Tungara frog calling courtesy of Brian Gratwicke on Flickr. License: CC BY-NC.

Understanding Number

- "animals, young infants, and adult humans possess a biologically determined, domain-specific representation of number"

- "a specific neural substrate, located in the left and right intraparietal area, is associated with knowledge of numbers and their relations ('number sense'). The number domain is a prime example where strong evidence points to an evolutionary endowment of abstract domain-specific knowledge in the brain because there are parallels between number processing in animals and humans."

–Dehaene, Dehaene-Lambertz & Cohen, TINS, 1998

What does "number sense" mean?

- Adults can represent large numerical magnitudes without verbal counting.

- The representations are approximate; discriminability of two numerosities depends on their ratio.

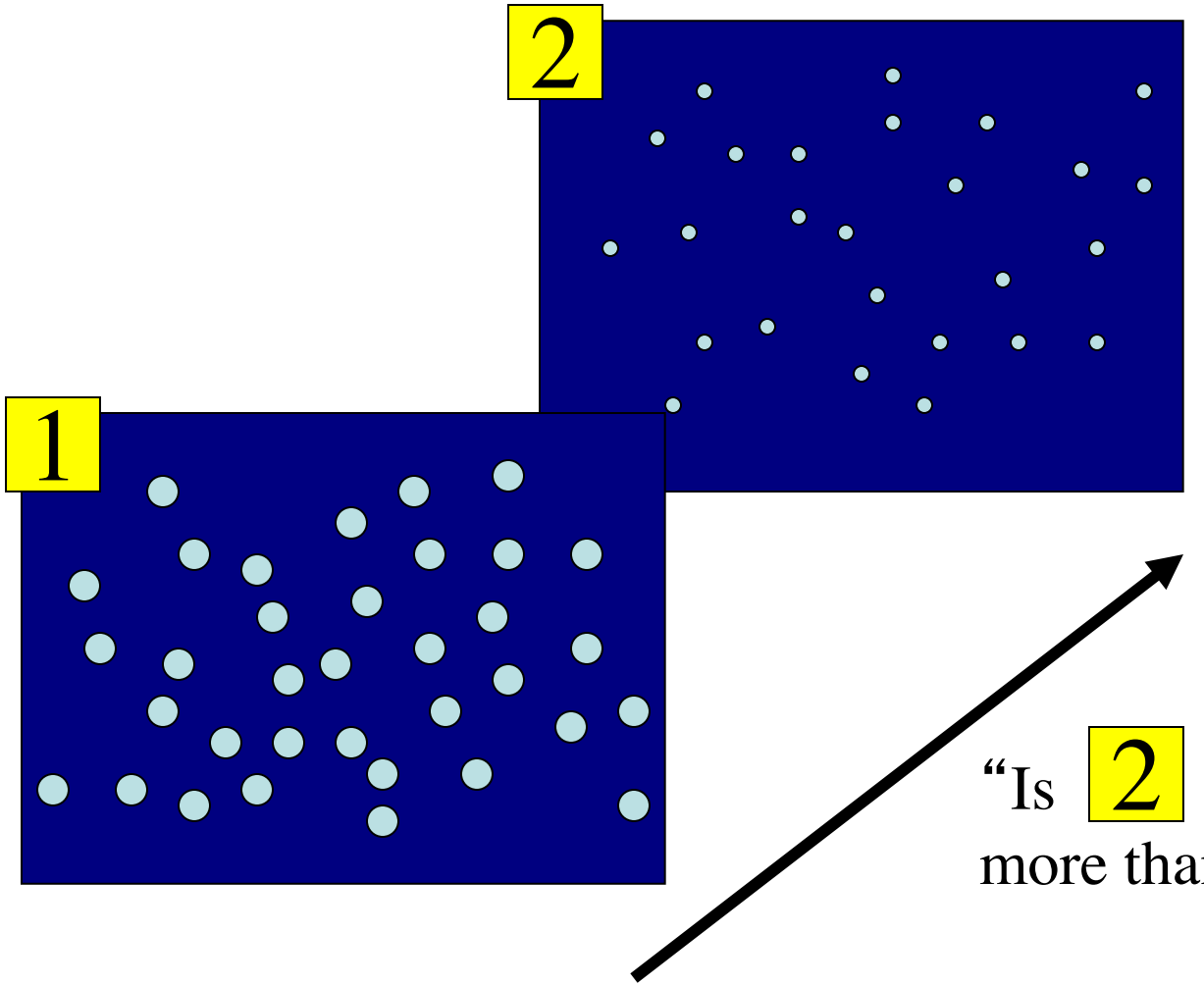
- The representations are abstract (vis-aud, space & time)

- The representations enter into arithmetic computations (e.g., addition).

For example……

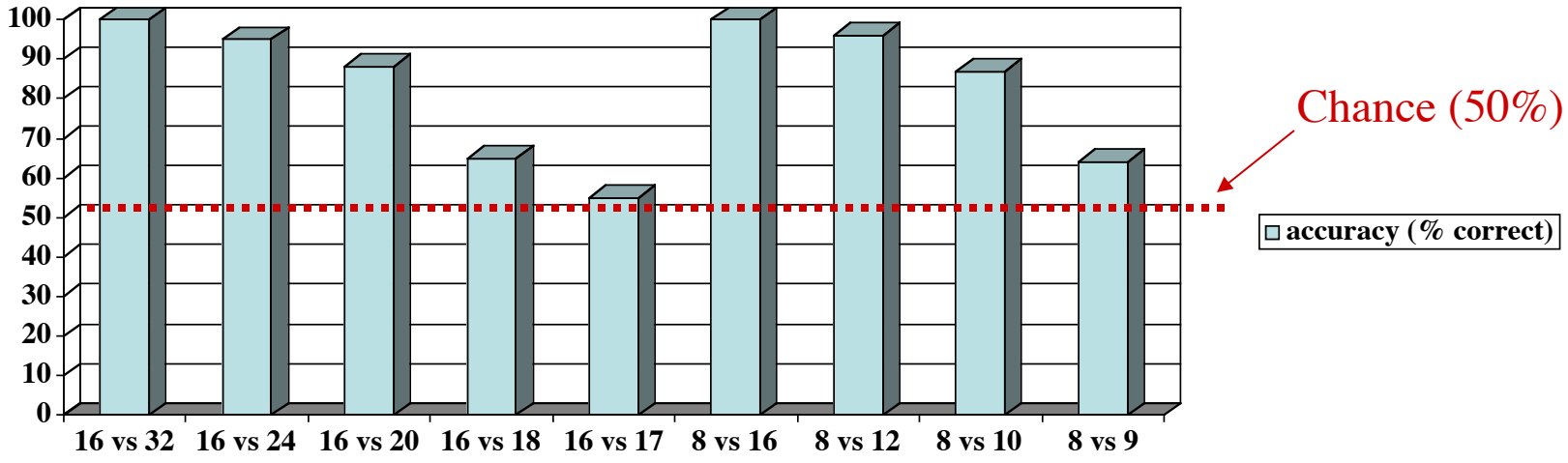
See video for the counting demo

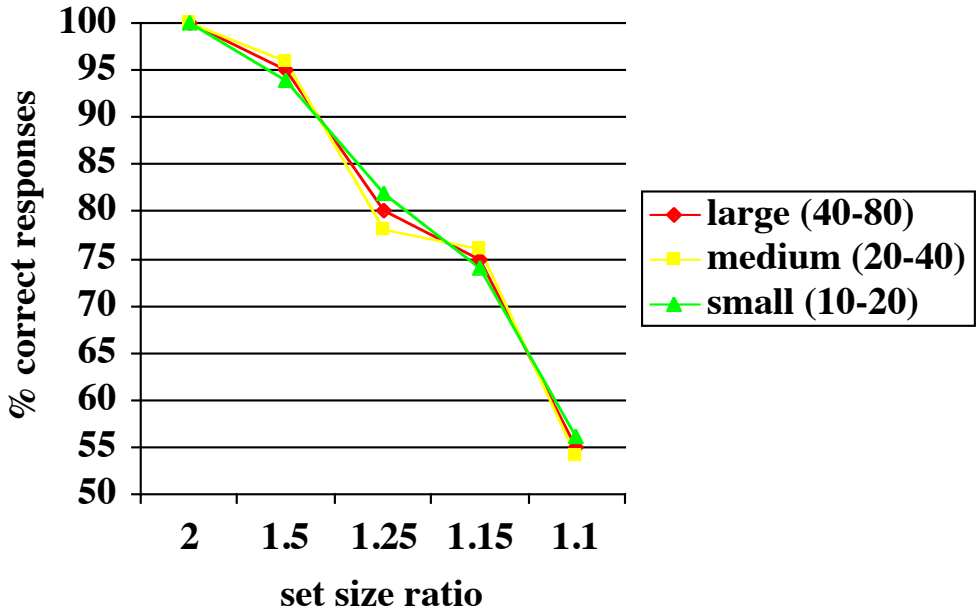
How accurate are adults' large number representations?



Barth, Kanwisher & Spelke, 2003

Numerosity discrimination by adults





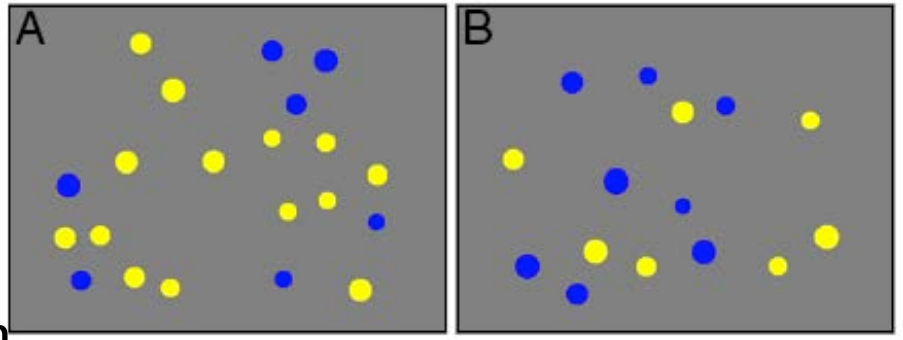
Weber's Law:
The discriminability of two numerosities depends on their *ratio* (not absolute diff).

Barth, Kanwisher & Spelke, 2003

What is this "Number Sense" all about and does it Matter?

"ANS" (approx num. system) test:
More yellow dots or blue dots?

A = easy, B = hard
Ratio = "Weber fraction"



- Big individual differences in "number sense"
- Developmental dyscalculia: specific deficit in ANS and other number tasks despite normal IQ.
- ANS ability develops slowly, best at 30
- Early ANS ability in kids predicts later math ability…

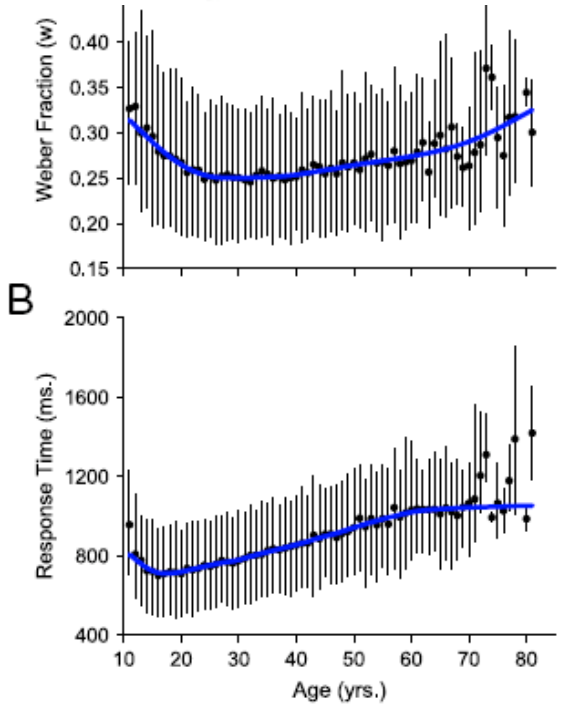
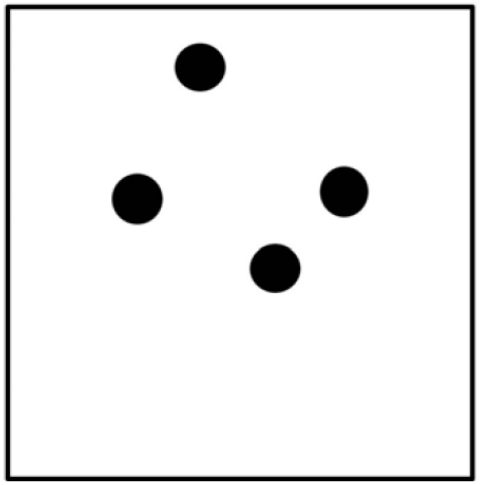


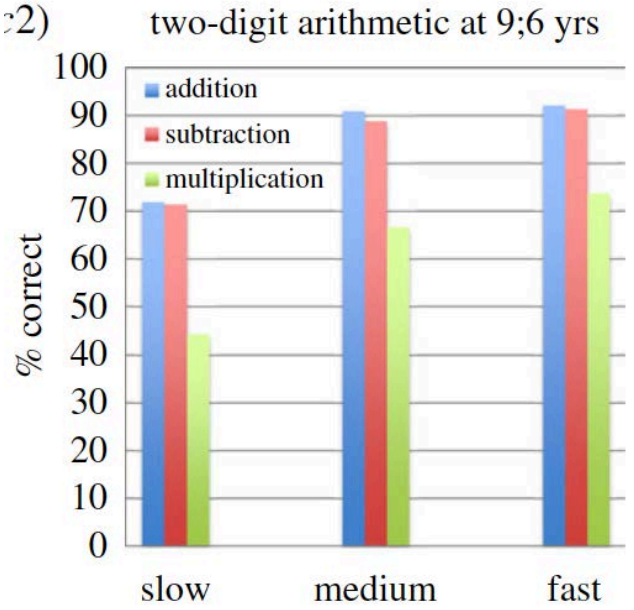
Fig. 3. Developmental changes. (*A* and *B*) *w* and RT (in ms) population trends computed as weighted cubic spline fit to *w* and RT means (blue lines) and interdecile ranges (10th to 90th; black vertical bars).

Longitudinal study of children from kindergarten to age 11

Reeve et al (2012), based on Butterworth et al (2018)



Question:
Is early number ability predictive of later arithmetic ability?



1. How many dots are there?
Answer as fast as possible.
Define slow, med, and fast groups in kindergarten.

2. Slow group (defined in kindergarten) does worse at arithmetic 4 years later.

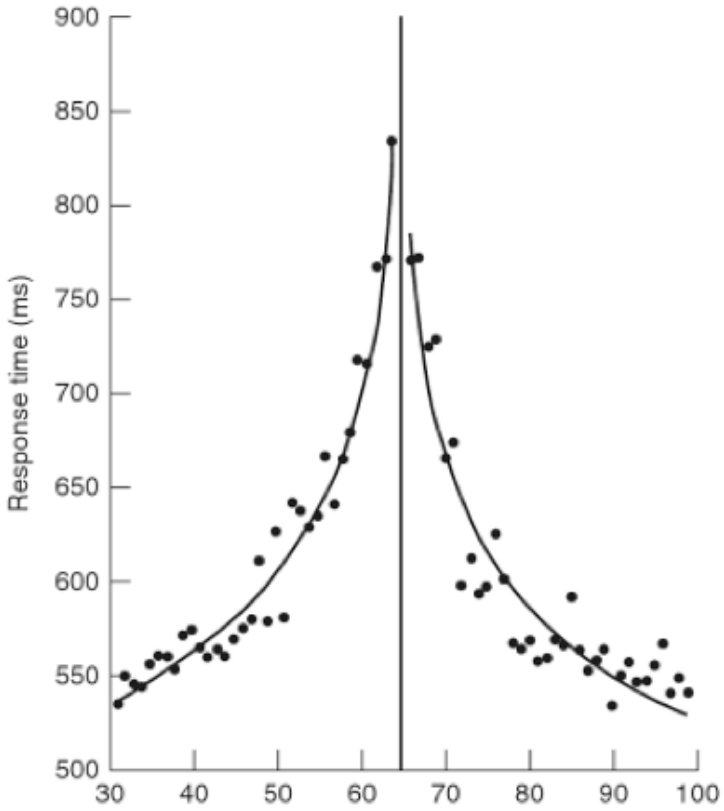
Speed of DE at kindergarten not associated with non-verbal intelligence (Raven's matrices), or ability to name digits or letters.

Suggests possible specific interventions for developmental dyscalculia.

Is ANS system engaged even in adults doing numerical tasks?

See video for number demo

Approx Number System Recruited for Symbolic (Arabic) Numbers



RT to decide if this (Arabic) number is greater or less than 65.

Suggests that understanding symbolic numbers taps into the same analogue magnitude system.

FIGURE 3.4. How long does it take to compare two numbers? Thirty-five adult volunteers classified all two-digit Arabic numerals between 31 and 99 as being smaller or larger than 65, while their responses were timed to the nearest millisecond. Each black dot shows the average response time to a given number. Responses become increasingly slow as the target numeral gets closer to 65: the distance effect.
(Data from Dehaene, Dupoux and Mehler 1990.)

What does "number sense" mean?

✓ • Adults can represent large numerical magnitudes without verbal counting.

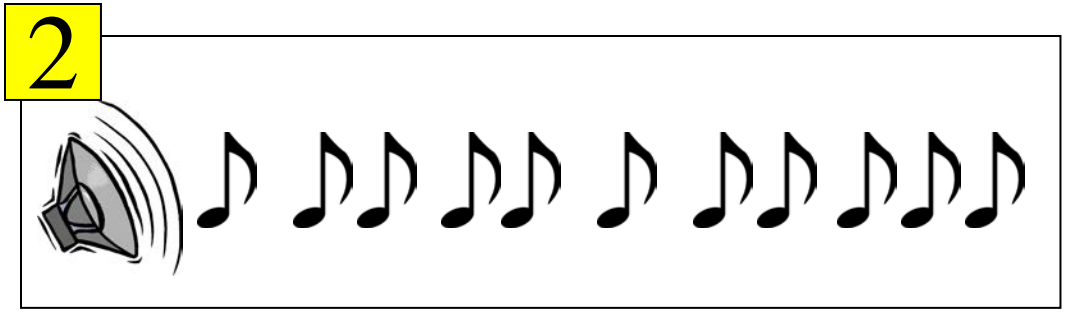
✓ • The representations are approximate; discriminability of two numerosities depends on their ratio.

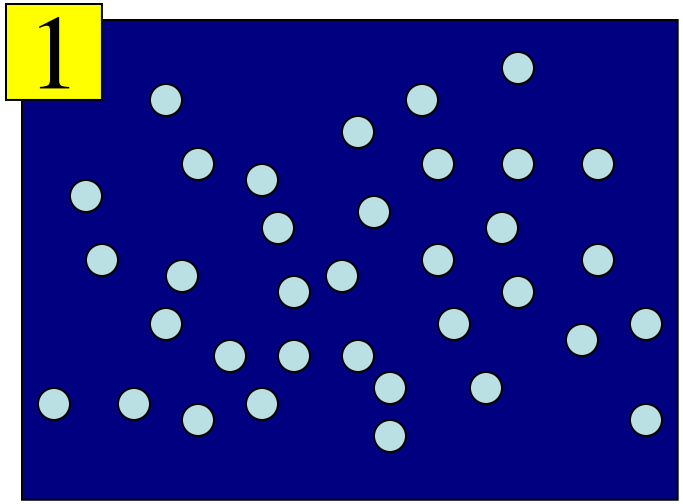
✓ • The representations are not based on continuous quantities like *area*, but rather on discrete *number*.

? • The representations are abstract.

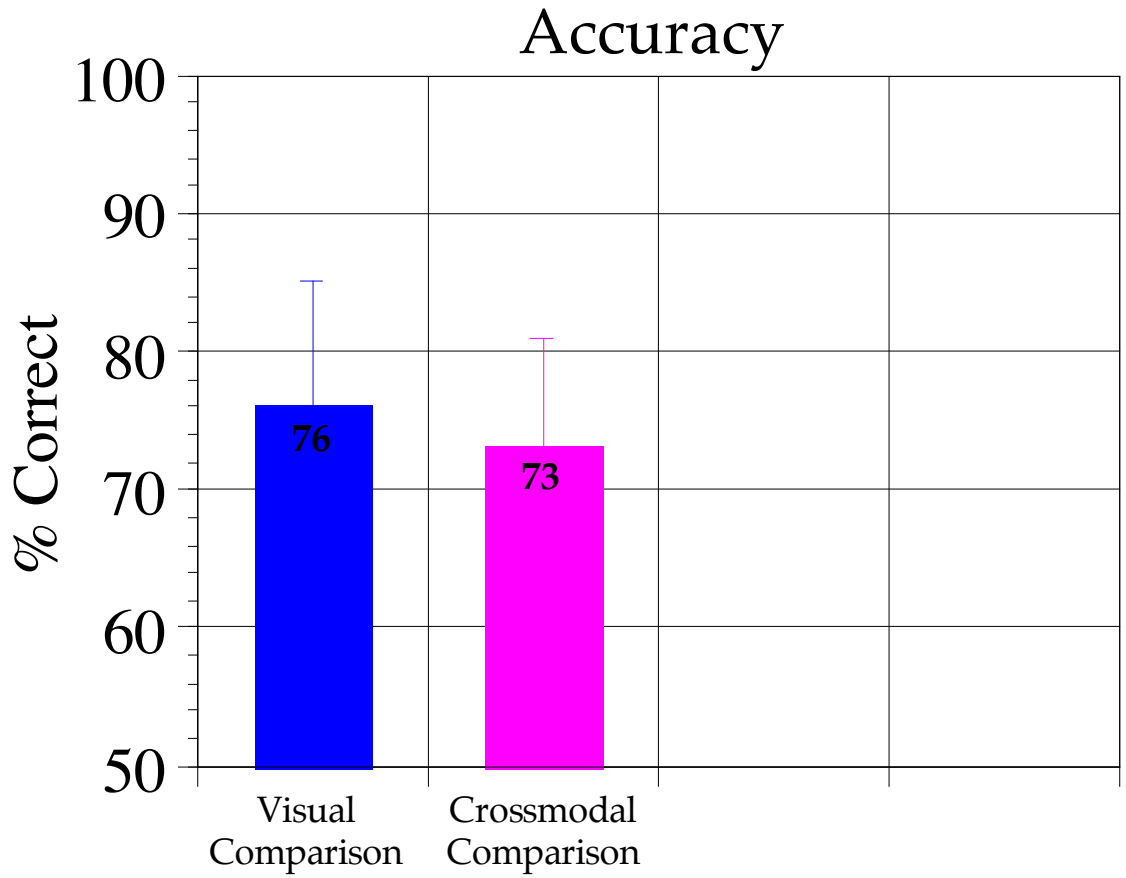
? • The representations enter into arithmetic computations (addition).

How abstract are adults' large number representations?



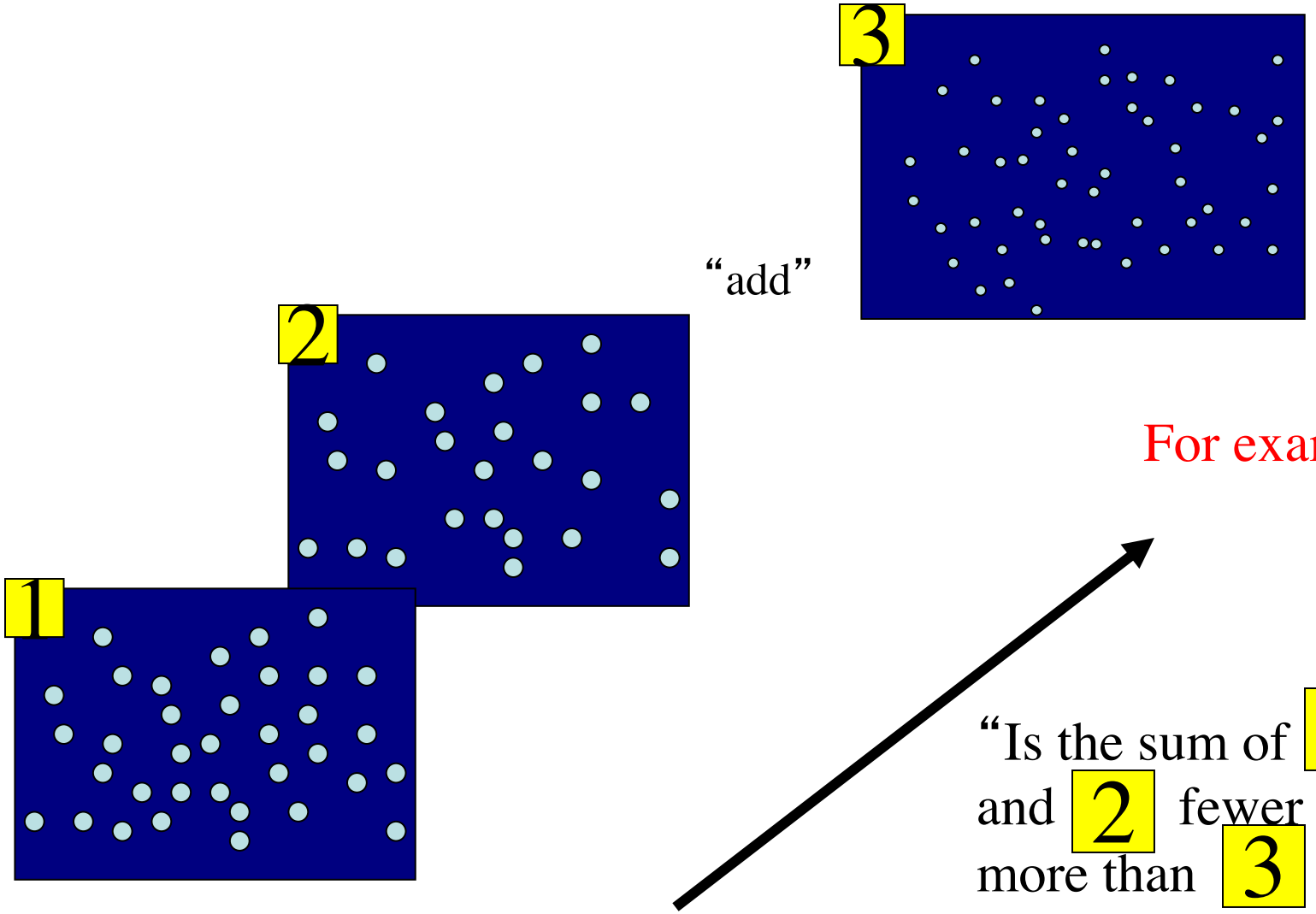


"Is 2 fewer or more than 1 ?"



Cross-modal comparisons are almost as accurate as comparisons within the visual modality alone.

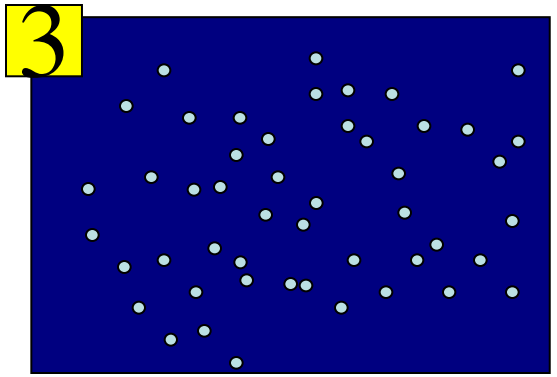
What can adults do with these large number representations?
Addition of visual arrays



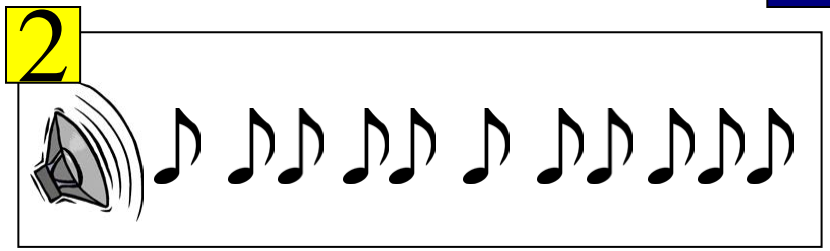
For example…

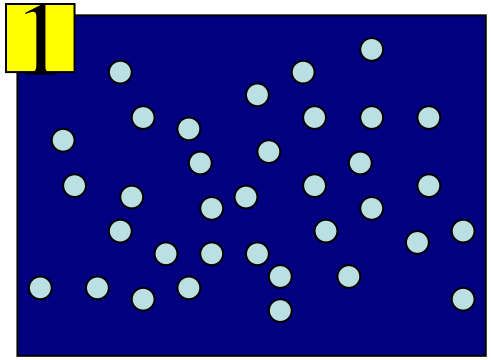
"Is the sum of 1 and 2 fewer or more than 3 ?"

What can adults do with these large number representations?
Cross-modal addition



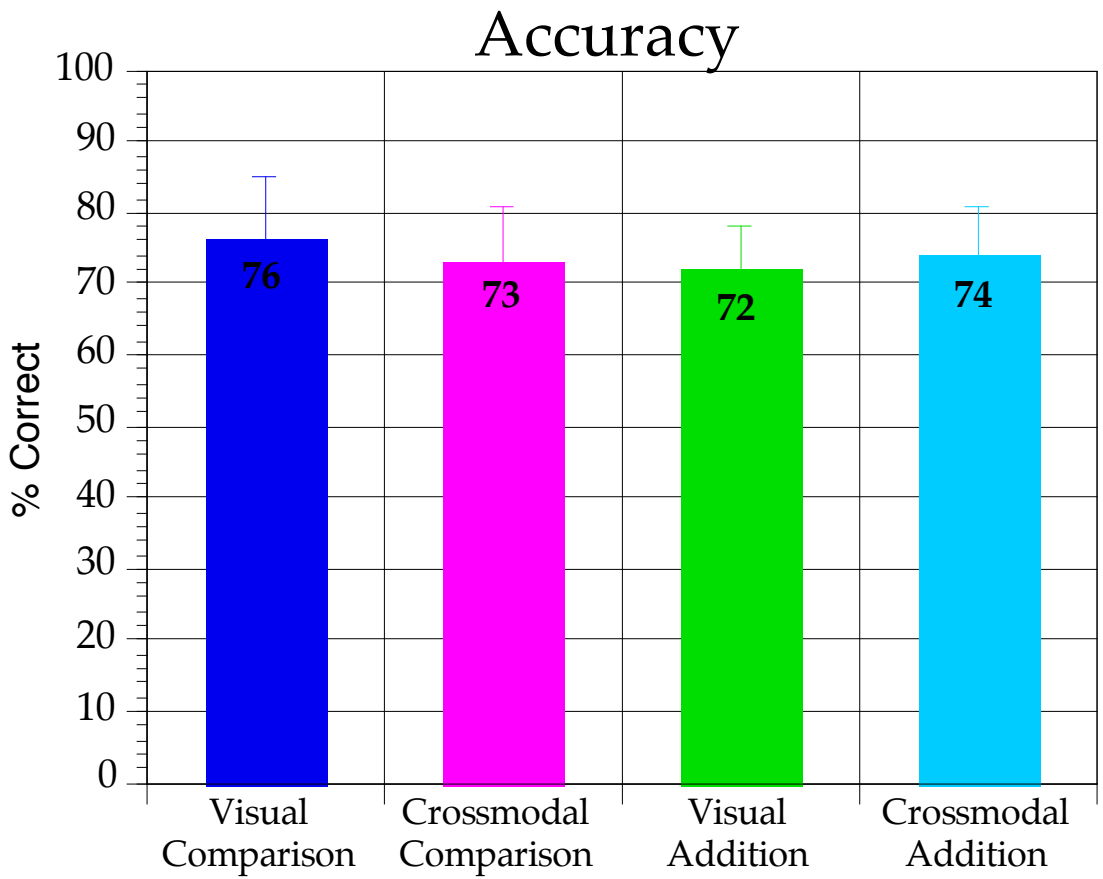
"add"





"Is the sum of 1 and 2 fewer or more than 3 ?"

Nonsymbolic Comparison and Addition



Barth (2001)

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✓ • The representations enter into arithmetic computations (addition).

But: the people in these studies have spent years learning and using formal arithmetic. Are these abilities exist innate? Do animals have them?

Is an Abstract Concept of Number Innate?

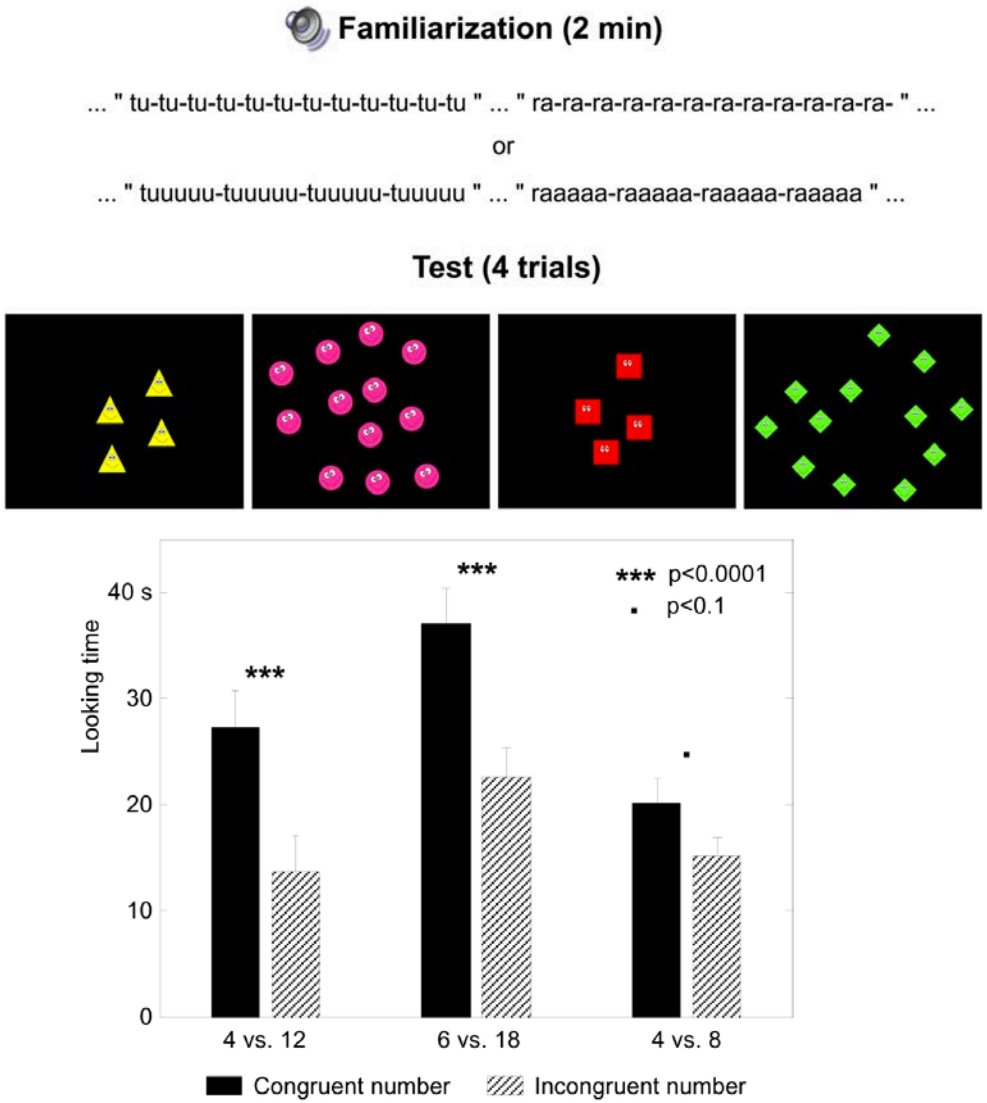
Izard et al PNAS 2009

Show 4-day-old newborns a number (auditory sequence); test for matching (visual array).

Newborns match number abstract to modality and space/time.

Shows ratio dependence. (Required ratio for discrim reduces with age.)

What about animals? Meet Mercury the macaw...



The Honeybee:

1 million neurons(vs 75 million for a mouse, 100 billion for human).

Honey bee image courtesy of Renee Grayson on Flickr. License: CC BY.

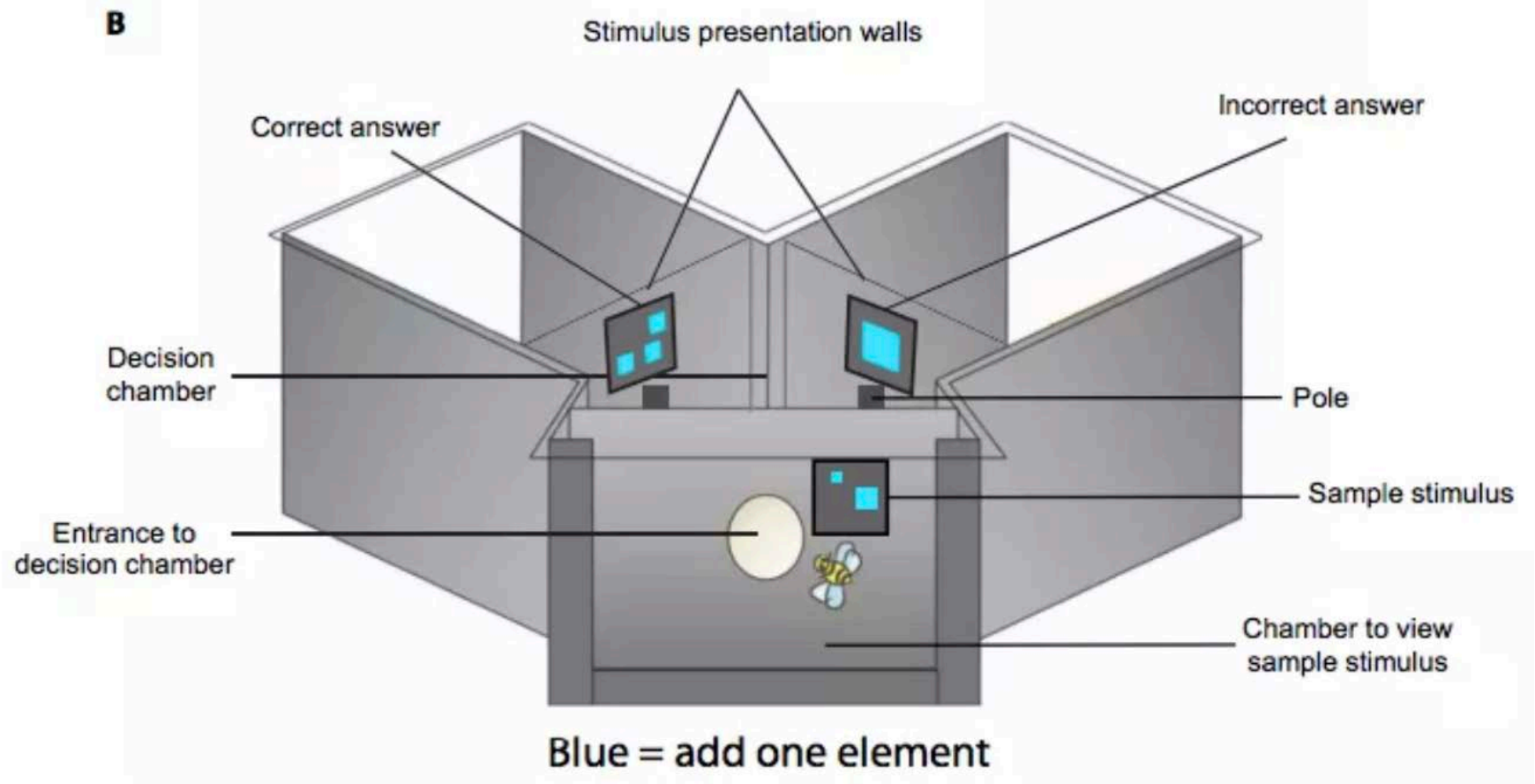
Bees' and humans' last common evolutionary ancestor lived more than 600 million years ago.

And yet...

Bees can do simple arithmetic!

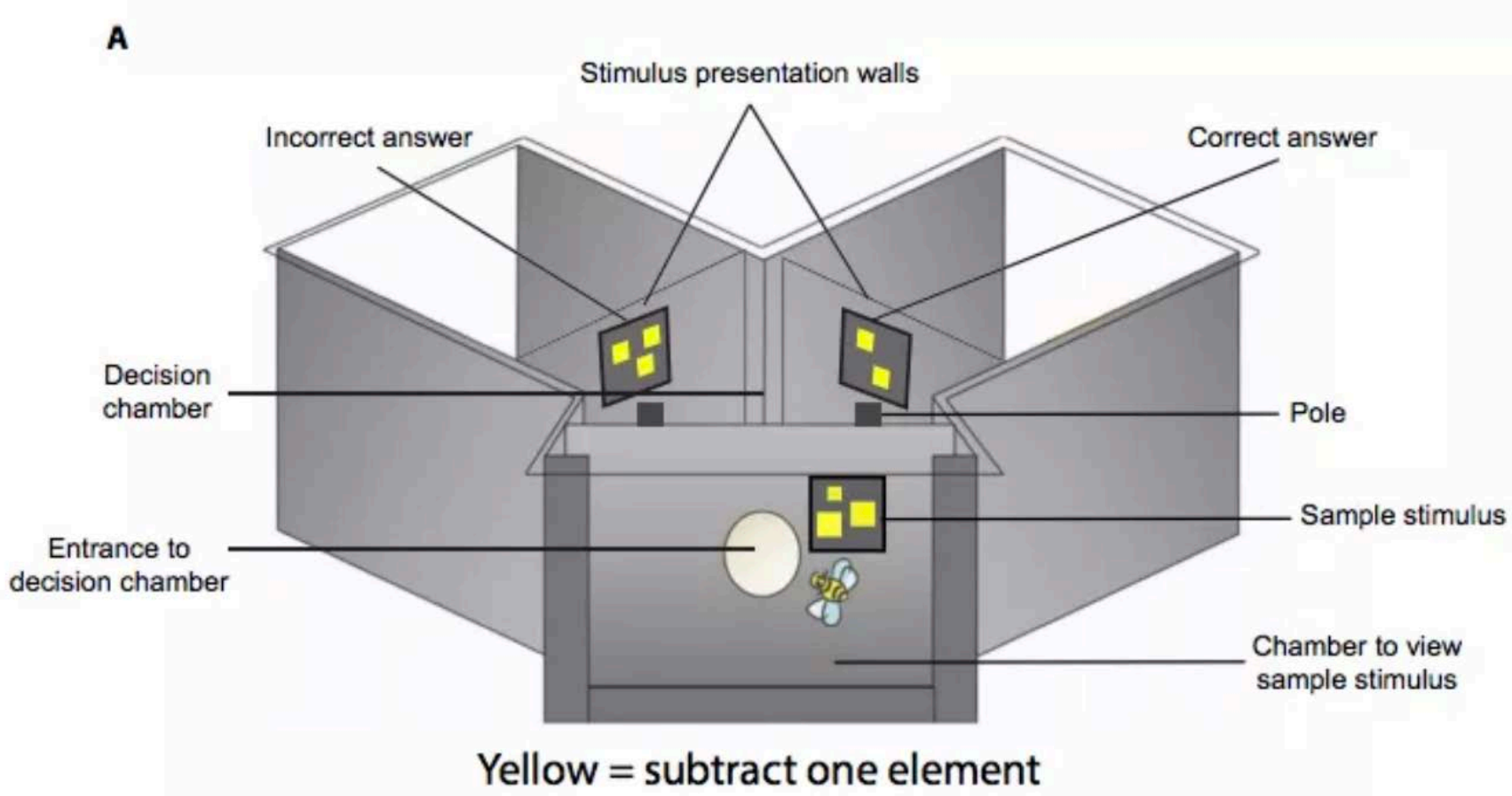
Train bees:

If the shapes in the first chamber are blue, bees should add one to get sugar reward. Bee enters decision chamber and indicates choice by landing on pole next to correct answer. If they chose incorrectly, they were punished with acrid quinine.



Howard et al, 2019

When the original shapes were yellow, the bees were rewarded for subtracting by 1, and punished for choosing the higher number.



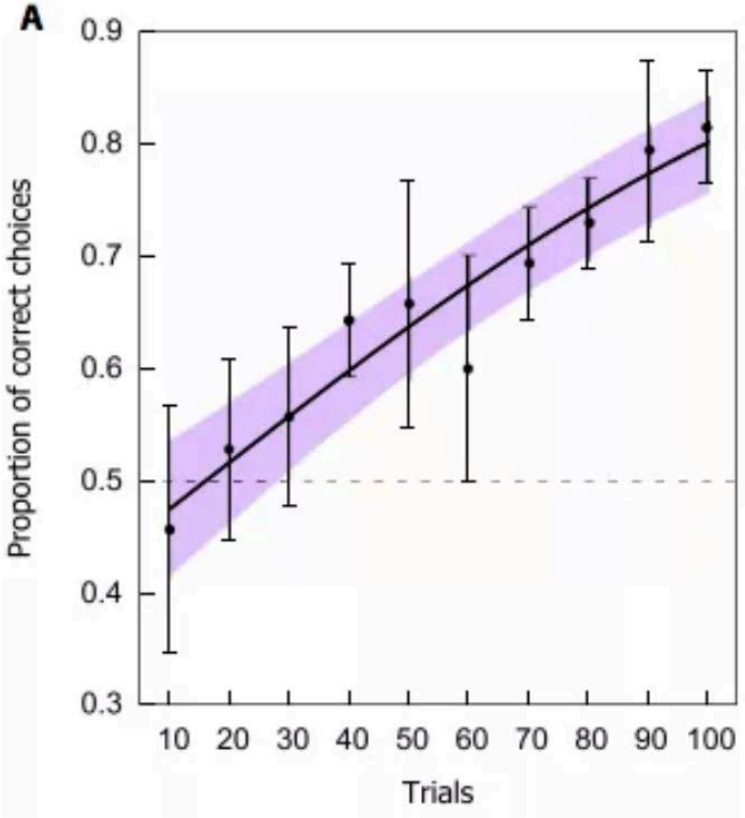
Total surface area is balanced.

Note bee has to remember the sample number to do the task.

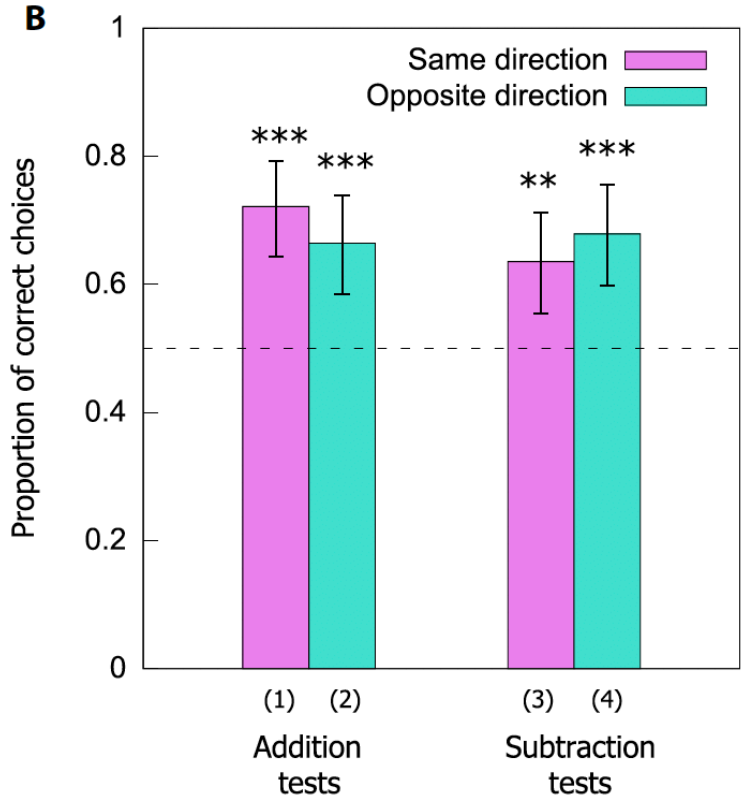
So, how do the bees do?

Howard et al, 2019

Over the course of 100 training trials, the bees got better at the task:



Now test with new numbers and shapes:



Is the bee just looking for more versus less?

No! Pink bars show performance when both options are more, or both less.

Cool. But what about even more abstract math?

Howard et al, 2019

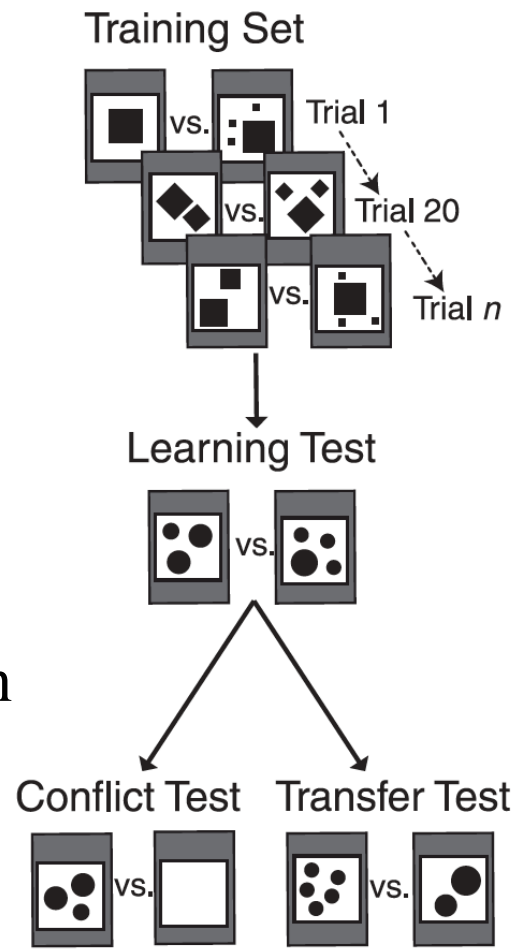
Do bees understand *zero*? Yes!

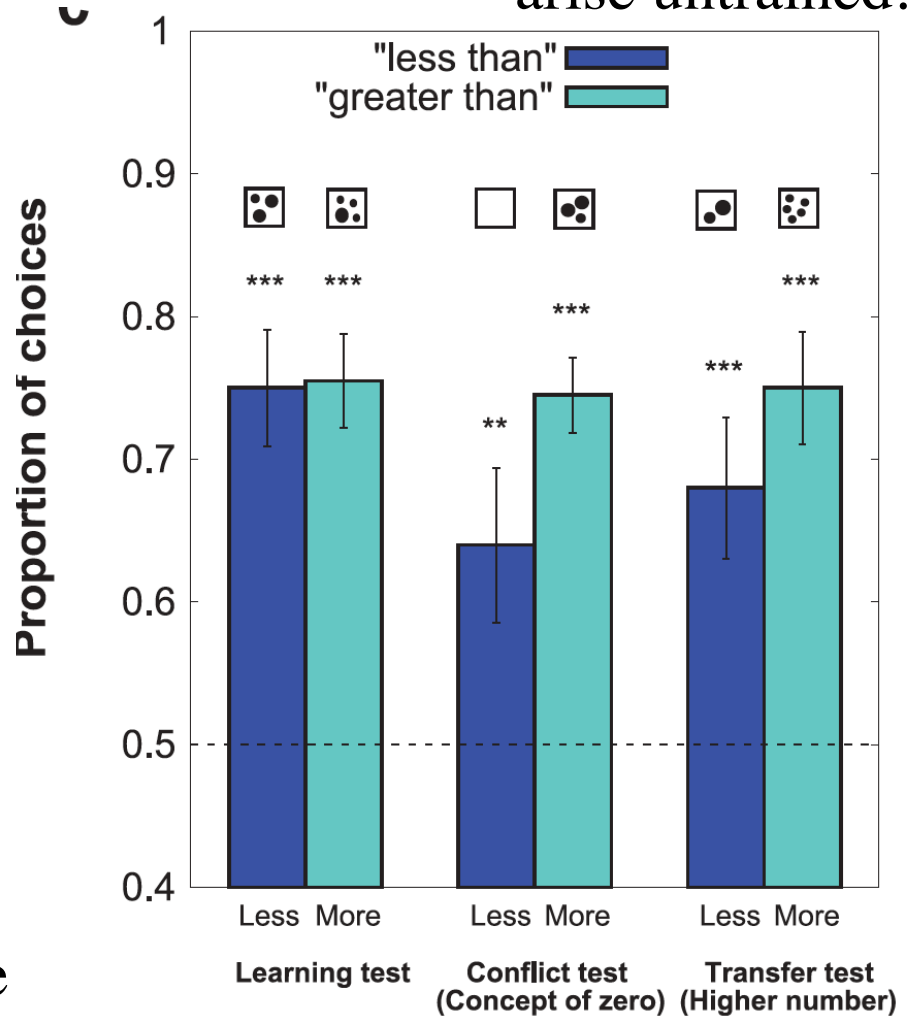
Train bees: "less than" or "greater than"
1-4 items

even addition can arise untrained...

Then test without rein-forcement:

Then test generalization





Test ZERO! test zero! test diff range

Howard et al, 2018

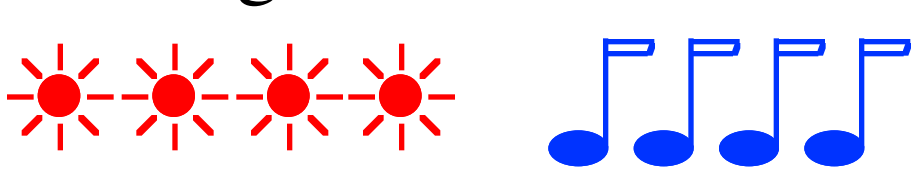
Church & Meck:

spontaneous addition and abstract number in rats

Training phase:

If 2 lights or 2 sounds press "2" lever

If 4 lights or 4 sounds press "4" lever

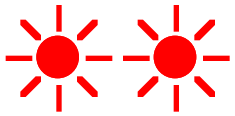


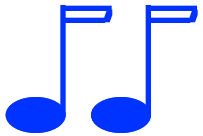
Church & Meck:

spontaneous addition and abstract number in rats

Testing phase:

Present 2 lights AND 2 sounds





Rats press the "4" lever:
spontaneous abstraction across modalities!

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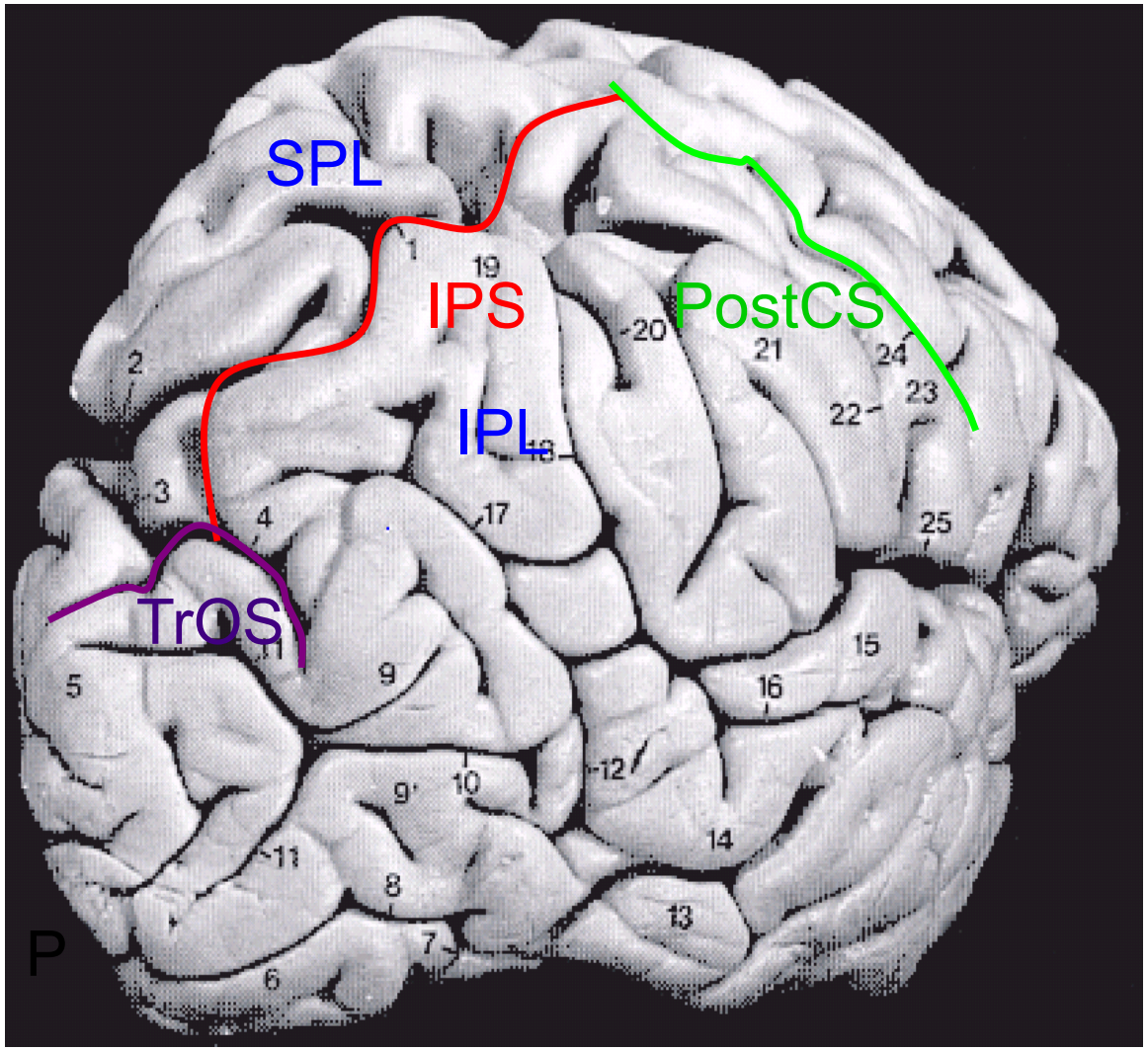
But: the people in these studies have spent years learning and using formal arithmetic. Do these abilities exist in infants? Animals? *YES!*

They are part of our basic cognitive machinery.

How are they implemented in the brain?

HUMAN PARIETAL CORTEX

Basic Neuroanatomy



Intraparietal sulcus (IPS)
divides superior (SPL) and inferior (IPL) parietal lobules

Neuropsychological Studies

Lemer, Dehaene, Spelke, Cohen (2003):

- One "acalculic" patient :
  - Left parietal lobe damage
  - Bad at approximation
  - More impaired on subtraction than multiplication (7-5 vs 7x5)
- Another "acalculic" patient :
  - left temporal damage
  - Intact approximation
  - More impaired at multiplication than subtraction

ANS deficit,
parietal damage.

- Taken together, these two patients are a……..???
- Why might subtraction and multiplication differ?

Just a few patients.
What can we learn from fMRI?

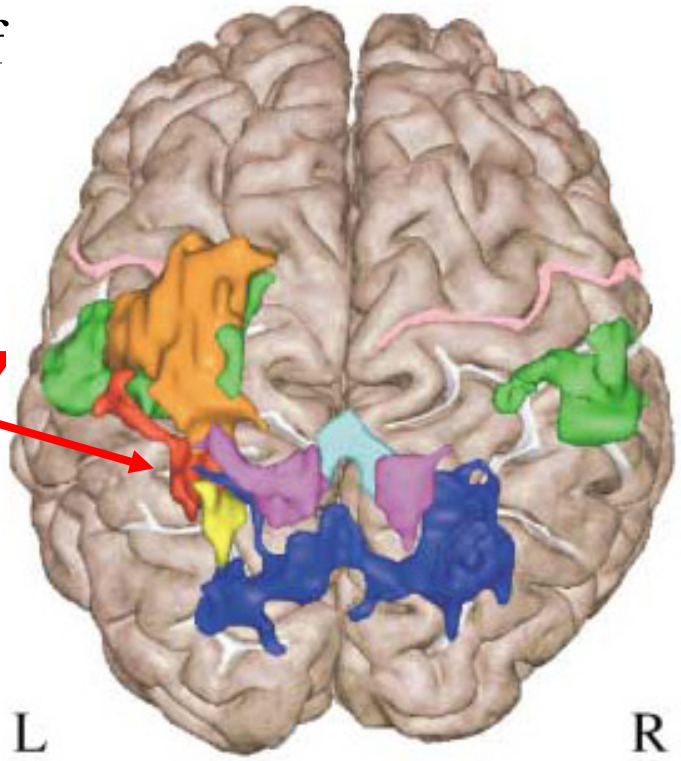
hIPS: The Locus of the Approximate Number System?

Simon et al., 2002

Claim: this parietal region is the locus of the approximate number system.

"hIPS"

It is involved specifically in number, not all the other functions tested here.



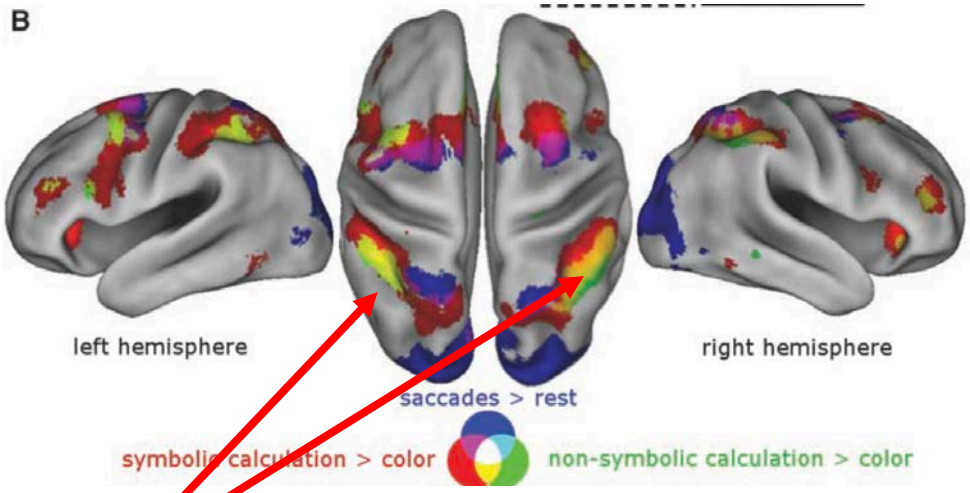
Annotated brain image © unknown. This content is excluded from our Creative Commons license, see https://ocw.mit.edu/fairuse.

Really?
These same regions long implicated in representing *spatial location*.

Alternate View:
No specific brain region for discrete number per se.
Instead, a common region for processing magnitude of almost any dimension
building upon space
e.g. the number line!

Grasping only | Saccades only | Calculation only | Attention only
Manual tasks | Visuo-spatial tasks | Calculation, Saccades and Language

Knops et al (2009) Assign Reading



"hIPS"

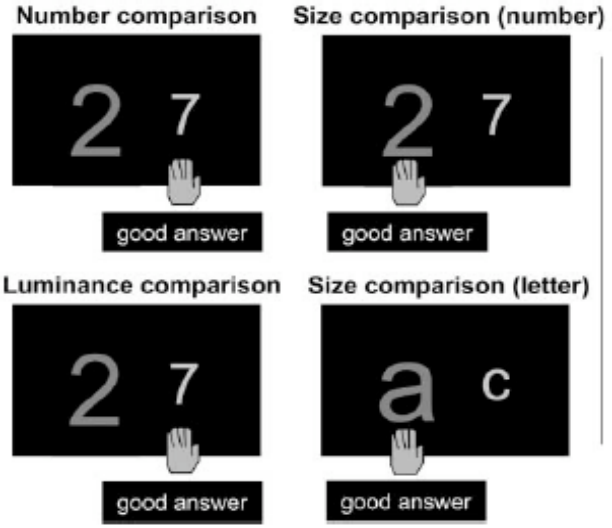
Do number representations share neural machinery with the spatial representations (aka the number line) - attention/eye movement system?

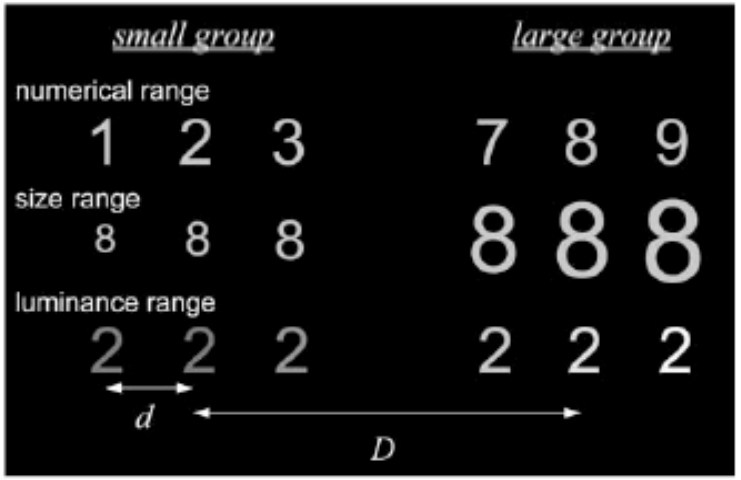
Key finding: train a classifier on rightward vs leftward eye movements > successful cross-classification of addition versus subtraction.
Evidence for common representations, aka the mental number line.

Brain Regions for Comparing Number, Size, & Brightness

Pinel et al (2004)

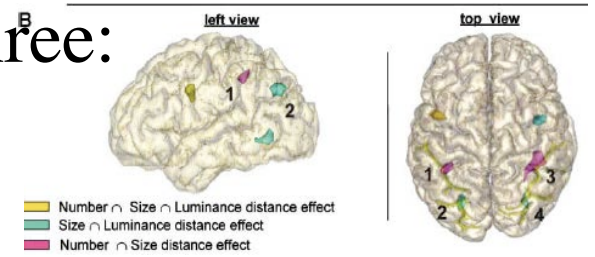
- Task: which item is larger (in number or size), or brighter?
  Contrast: Small (hard) > large (difficult) differences, e.g. (2 -3) vs (3-7) (3





- Result: Similar regions for all three:

> In our study, no region appears specific for a given dimension, in the sense of exhibiting a significantly greater distance effect for one dimension than for the others. Nevertheless, the location of activation peaks



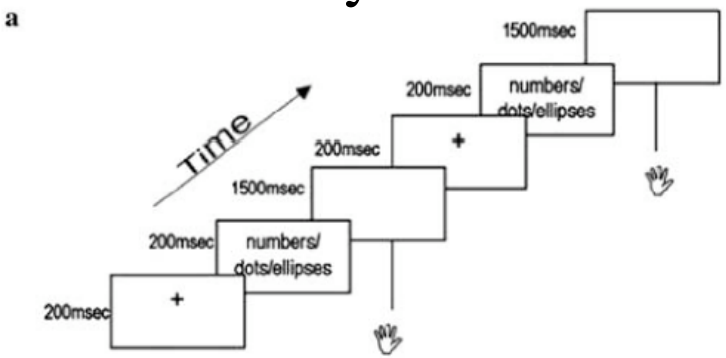
"number and size engage a common parietal spatial code"

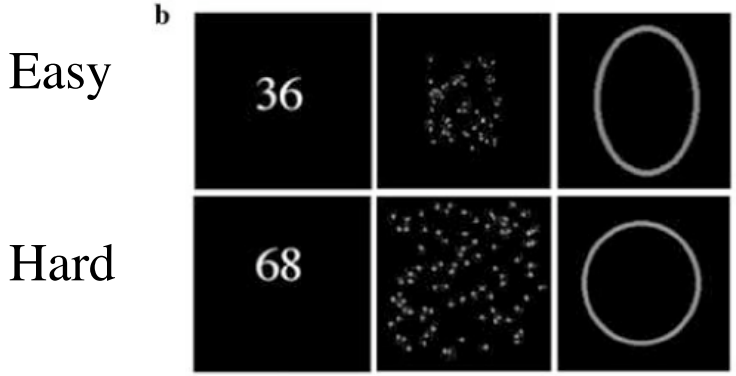
- Not just discrete number, but "Magnitude" Or, just any difficulty?

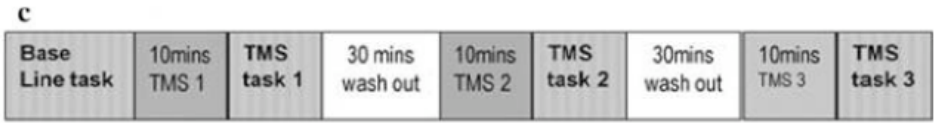
TMS to IPS: Is IPS specific for number?

Cappelletti et al (2007)

Both symbolic and nonsymbolic number?







Offline TMS to left IPS

- Disrupts magnitude tasks on numbers and dots
- *Does not disrupt horiz/vert ellipse judgment*

So:

Some evidence for specificity of role of left IPS in symbolic and nonsymbolic number, not just any difficulty.

Task: greater or less than 65?

Horizontal or vertical?

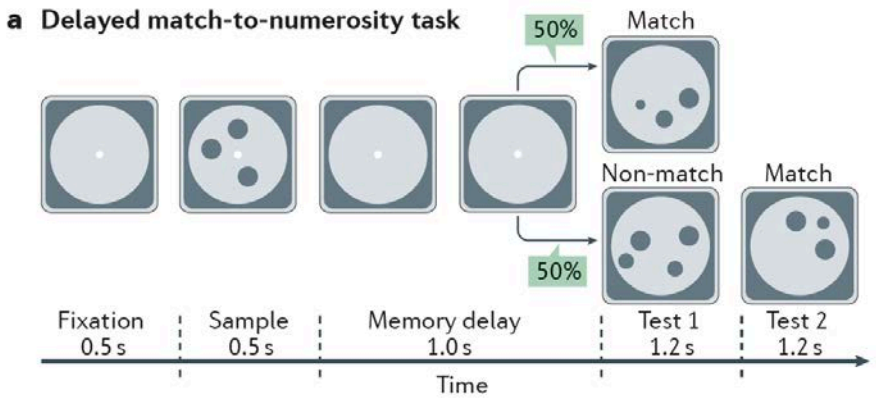
One easy version, one hard version of each.

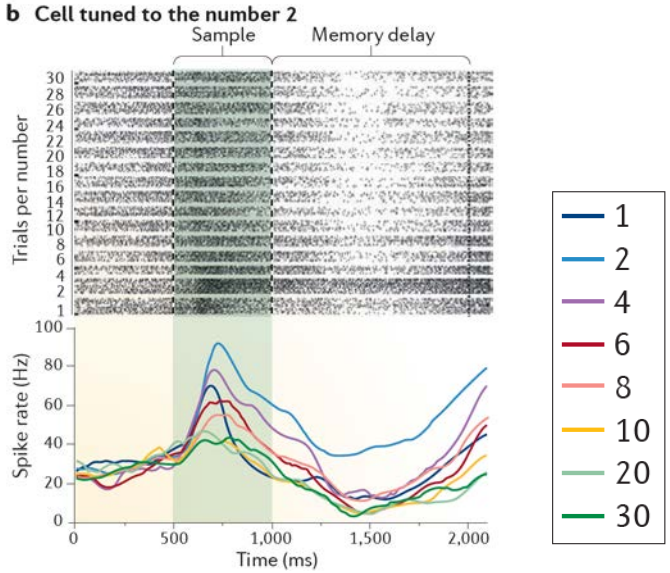
What are the actual neurons doing in here?

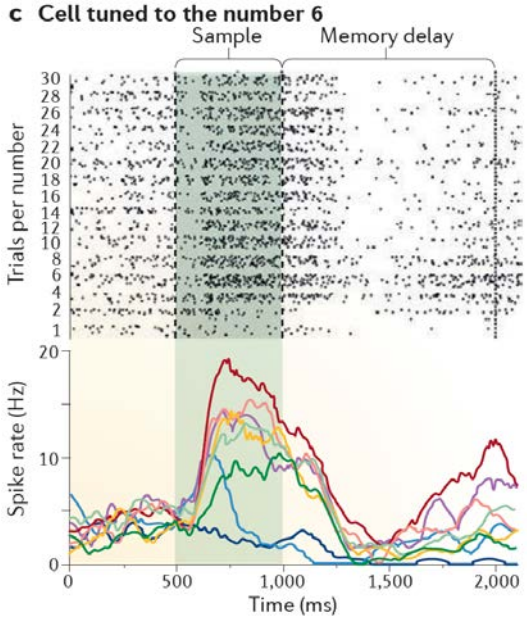
How is number represented in neurons?

1. Train monkey to do number task:

2. Record from neurons in parietal & frontal cortex.







from Nieder, NRN2016

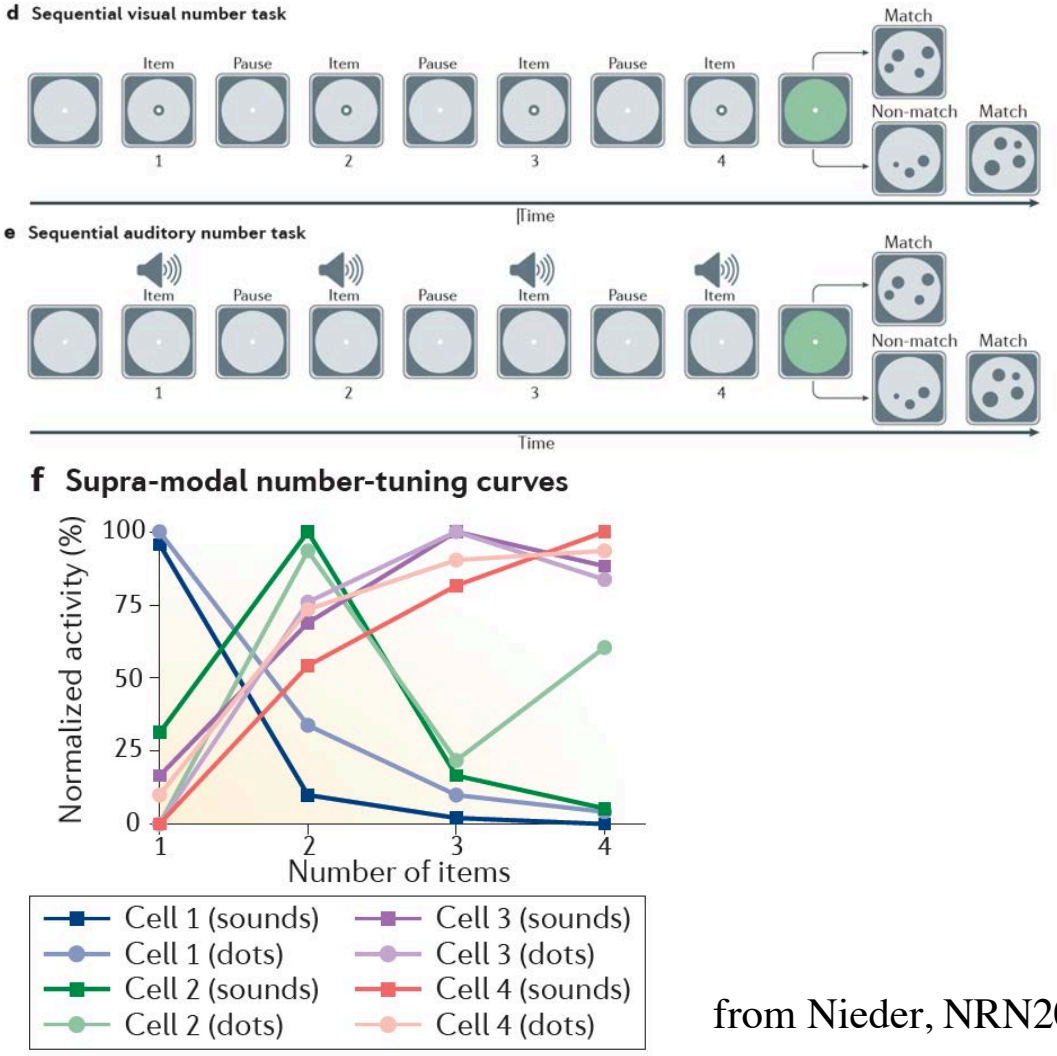
Source: Nieder, A. The neuronal code for number. Nat Rev Neurosci **17**, 366–382 (2016). https://doi.org/10.1038/nrn.2016.40

Cool! But how abstract?

How abstractly do these neurons represent number?

1. Over both space & time? YES!

2. Over modality (vis and aud)? YES!



from Nieder, NRN2016

Find supramodal number neurons.

But these monkeys were *trained* on number tasks! Does this matter?

Apparently not in parietal regions (Viswanathan & Nieder 2013).

Summary/Conclusion about the Mind & Brain's Approximate Number System

Approximate magnitude system

- shared with animals and newborns
- follows Weber's Law
- abstract to object features, modality, symbolic/nonsymbolic
- Big individual diffs in ANS system
  - developmental dyscalculia (with normal IQ)
- Childhood ANS ability predicts later math ability
- hIPS is key cortical region,
  - not just for numerical magnitude but space/time/luminance
  - ??? General magnitude, or any difficult process?
- Number neurons tuned to specific numbers (!)
  - abstracted from just size/continuous quantity

We can give the last word to Stan Dehaene

Understanding Number

"the brain treats number like a specific category of knowledge requiring its own neurological apparatus in the parietal lobe [hIPS]…..

When it comes to subtler distinctions such as number versus length, space, or time, however, the specificity of hIPS vanishes. *No part of hIPS appears to be involved in numerical computations alone*."

"the human brain is neither anisotropic "white paper", where all regions are equivalent, nor a neat arrangement of tightly specialized and well-separated modules."

From Dehaene (2011), *The Number Sense*

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