RUMEN DANGOVSKI: Hi, everybody. I'm Rumen, and today I'll talk about three things: optical trapping, the Boltzmann constant, and Brownian motion. The goal of the lab is to extract Boltzmann's constant out of Brownian motion and there are two key components to think about.

First one is the Boltzmann's constant, which is prevalent in different types of science. For example, we can have it in biophysics where people use the Boltzmann's constant to try to understand forces in a cellular level. We have it in thermodynamics with the famous equipartition theorem. How does this relate to the Brownian motion? The key thing is to think about length scales. Brownian motion is relevant in terms of microns. And at the same time, if you look at a cellular level, for example, if we look at our hair, we have micron-sized hair.

All right. So there are different ways to measure the Boltzmann's constant. Some people measure the speed of sound in argon gas, others do optical trapping in air. We will do slightly different optical trapping, as you will see, but the consensus among the scientific community is that it is challenging. Our plan is to prepare Brownian particles and control their Brownian motion. We take spherical glass beads of diameter of 3.2 microns and the main tool is to concentrate a highly-focused lasers on top of these beads. There's interesting physics going on: light carries momentum thus it generates force. We have that the net gradient force opposes the motion of the beam while the net scattering force goes along the motion of the beam. And when these two forces balance each other, we have a bead that is at the center. When you push this bead a little bit to the left or to the right, then we have that the gradient forces are pulling me back in the center and essentially we observe a simple harmonic motion.

In a more concrete example, what we did is we took samples and we confined everything into a two-dimensional plane. This is very important. We have two directions: the x direction and the y direction for the beads. We put them into water and a source of Brownian motion comes from the collisions between our bead with the molecules into the water. These are thermal collisions that generate Brownian motion. As you can see, this lonely bead has it's Brownian motion. What's interesting is when we shine light on top of a bead, we trap this bead and we can find the Brownian motion so it's feasible to measure the motion.

And the theory behind this is very beautiful. It's about the equipartition theorem, which relates the kinetic energy coming from the simple harmonic motion on the left hand side with the thermal energy due to the degrees of freedom. Now let us recall that we have a two-dimensional confinement so we have a direction in x and direction in y. So it means that in each of the directions, we have one degree of freedom, which is correlated with this equipartition theorem. Now an interesting thing about the statistical motion of the molecules is that we have the simple harmonic motion. However, the things are moving into water so there is a drag force that dominates. So we simplified the left hand side. What is very interesting is the f factor, which comes from the collisions between the bead with the molecules, this generates forcing and driving of the simple harmonic motion.

I would like to point out one thing about the scales of the forces. We have piconewtons, which is relevant for optical trapping and for Brownian motion. This is the first observation that we did, and actually Einstein did this observation a long time ago. He observed the white noise-- essentially the collisions-- they generate uncorrelated forcing. And we can think of it as something that is not biased with any distribution, it's just uniform. As you can see on these slides, we have the position plotted in terms of time and it exhibits a uniform distribution of the spectrum. Another thing that I would like to point out is to look at this plot of fluctuations in x and y, and the power which is linear with the current of the lasers. As you can see, as the power becomes big, this means that we are trapping more so we have less fluctuations, which is something as expected.

All right, so let's figure out what we want to do with this lab. We have the equipartition theorem and essentially we have three components that we would like to measure in order to extract Kb. We need to find the fluctuations, which I just presented to you. We need to find a stiffness coefficient, alpha, which is related to the spring constant of motion. And then we need to measure the temperature, t. The apparatus that we use has two main components. The two components are concerned with two types of light that we use in our experiment. We use a laser that shines on top of the confined two-dimensional samples and tries to trap a bead. The scattered light from the laser goes into a QPD-- a quadrant photo detector-- which is an ultra-fast camera that manages to quickly digitalize the content and give us the position of the scattered light. So when we trap the bead, we know where it is by observing the feedback from the QPD.

The other interesting part of the apparatus is the LED light, which illuminates the sample and then it brings it to the CCD camera, and this is very important. So the CCD camera is very slow. It cannot measure Brownian motion. What it can do is it can tell us where are the beads.

It can allow us to look at the water and find the beads we want to trap. So these two components in combination are crucial for the success of our research. There is interesting electronics coming behind this. Essentially, the signals from the QPD and from the stage position where we put our sample are given in terms of volts.

So a natural question that arises is how are we going to remember what is important? There are two important things that we like to keep track of, two positions. The first one is the position of the stage that gives us a relative point of consideration. And the second one is the position of the QPD, which, as you can recall, it measures the place of the bead that we have trapped. These inputs are given in terms of volts, so the natural thing to do is to find the calibration that converts these volts into actual distances. Here you can see how we do this. We plot the stage x position-- we can plot the stage y, it's completely analogous to the QPD the positions here. And the conversion factor hides along the slopes here. As you can see, we have two different slopes here with different absolute values, so this type of measurement is prone to errors. How we approach this problem is that we fault by choosing more data points and trying to increase the statistic thus hopefully reducing the systematics of this measurement.

The second step is to start getting the components that we need in order to extract kb is to measure the stiffness coefficient alpha. Now the main idea is to take the equation of motion and to make a [? free entrance ?] from in order to get the positions in terms of frequencies. There is mathematics behind this and essentially, the power spectral distribution, and that's the distribution of this motion here that we expect obtains this form here. From where we can extract the characteristic frequency, F naught, and F naught gives us alpha, which is what we really need. This is a log log plot, as you can see here. Another thing I would like to mention is look at the proportionality here. As we increase the power, we also increase the stiffness coefficient alpha because we're trapped more closely, so it means that F naught has to increase. And indeed, when we increase the power, we are shifting F naught to the right.

Having all of these components, we can put this into the big picture. And the big picture is the extraction of kb over here. I showed you x squared, the fluctuations. I showed you alpha. We can measure the temperature, t, and then we can get kb. The essence of this measurement is to look at the inverse proportionality between fluctuations and the trapped stiffness. Then we can make a fit with a reasonable chi-squared probability, and from here we can extract kb. The result is presented on the slide. We also show you the measurement that we extract from literature. Our result is within 2 sigma of the accepted value of kb. Actually we're very close to one sigma from the accepted value of a kb.

What is driving this unfortunate outcome? The thing that drives this unfortunate outcome is the systematic errors on our picture. This is our starting error. This is the uncertainty in kb. And there are different factors that contribute to this. As you saw, the calibration is very difficult. We have error from the laser hitting the water and changing the temperature. We have systematic error of the electronics, which we safely ignore because the first two factors dominate this. Our electronics were very precise. And then we have the statistical uncertainties that we use which correspond to some error propagating tricks.

OK, now let's do our investigation. The first one is about the calibration. As you can see, all of these slopes should be valid, but they're actually not. So what we do is we take them into account, we average them, then we take more data points, we average again, and then we propagate errors in order to reduce the systematics. The second one is due to the heating of the laser. Essentially what happens is when the laser hits the water, it starts heating the vicinity. And this is unfortunate because yes, we can measure the room temperature by using the thermometer, but actual uncertainty on the temperature is much bigger because we have extra heating due to the laser. We tried diligently to avoid this by moving the laser constantly so that it doesn't stay in one place and heat up a lot, but it's very hard to quantify how exactly it heats up the water.

And the third one is concerned with our fit with the fluctuations in terms of the trapped stiffness. Initially, when we did the fit with the PSD method, we get a probability of chi squared equals zero. And then we quickly realized that essentially what we need to do is take into account the horizontal errors. And here what we do is we transform the horizontal errors into the vertical errors. And this thing we can do by using addition in quadrature and propagation of the errors, which gave us a reasonable realistic Chi squared of 0.33.

In conclusion, I'll start with limitations. As you saw, it's very hard to distinguish between systematic errors and statistical uncertainties. The reason for this is, as you saw, is that we are fighting with the systematics by introducing more statistics and everything mixes up in the propagation of errors. The second thing that was very difficult in this lab is that we need to move the laser constantly. So one of the people working on this lab has to keep track of where the laser is and whether you're trapping things that you may not want to trap. The third thing is the need to find better ways of calibration. And there are people who are actively working on this topic here. As you saw, the main source of error came from the calibration, actually. But the plus is that we can give a reasonable estimate to Boltzmann constant and at the same time, we can have a lot of fun while doing so.

I'd like to thank to my partner Emma. And I think collaboration is very important for JLab and more specifically for this particular experiment. This experiment is impossible to be done without a partner because it's very difficult. It's very difficult to keep track of where you want to put the laser and also what is going on around the laser. While one of the people is moving the laser, the other one should be looking at the CCD camera and telling where are we going and what are we actually trapping? What do we want to avoid? Like we don't want these guys in our picture. I would also like to thank the staff of this class for their useful feedback and their valuable help. And finally, for your attention.

[APPLAUSE]

AUDIENCE: So in this experiment, we found it useful to consider the trap being elliptical rather than circular. And so it strengthens alpha, being different in the x and y directions, did you do that in your calculation?

RUMEN DANGOVSKI: Yeah, so we have a lot of this should be trapped stiffness versus this is the current. And here, actually I haven't shown that. The trapped stiffness differs whether you are looking in the x direction or the y direction. The actual measurements are analogous because the mathematics is the same. But as you can see here, we have different data points for the two cases. We account this into our considerations, yes. Yes?

AUDIENCE: So you mentioned earlier that [INAUDIBLE] with air or in air [INAUDIBLE]. Is there any benefit-- or what's the main difference between [INAUDIBLE]?

RUMEN DANGOVSKI: Everything boils down to this consideration here. Let me just find my explanation. It boils down to the viscosity that dominates. So in our case, we have the viscosity of water that dominates. When you look in different mediums, you might have different types of viscosities which would change the motions. So I have the paper, I didn't actually read through the whole details of how exactly the mathematics changes, probably does. Also probably another issues that may not arise or may arise is the laser, like in this case. The laser heats up the water, but I don't know how exactly the laser would react with the air. Maybe it's going to heat up a little bit, but how is this heat going to be distributed in space? I'm not very knowledgeable of this as of now.

PROFESSOR: Any other questions?

AUDIENCE: Right there, what does it say? Only one degree of freedom [INAUDIBLE]?

RUMEN DANGOVSKI: So the equipartition theorem, it breaks into components. It breaks into the component of x, where you have one degree of freedom, and it breaks into the component of y, where you have another degree of freedom. Each of these considerations is concerned only within one movement.

AUDIENCE: So you have two degrees of freedom?

RUMEN DANGOVSKI: OK, well, you can think about it in this way. We have two degrees of freedom in total, but in the actual directions that are useful for our considerations, we have only one degree of freedom.

AUDIENCE: You said you were using the 3.2 micron [INAUDIBLE]. There were other sizes available. It looks like some of your photographs from the [INAUDIBLE]. Did you try data from both the small beads and the big beads? Curious how they compared in quality.

RUMEN DANGOVSKI: This is 3.2 microns. Later on, you saw the one microns. I think our data is concerned with the 3.2 microns. My results are not related with the one micron. We just played with it and tried this. We also tried trapping cells from onions. It was very fun to play with, but unfortunately, we couldn't get any valuable quantitative results there. So even though it's quite a lot of fun, we have some clips, it's not worth for this presentation, which concentrates on the kb.

[APPLAUSE]