MITOCW | L22.4 Identical particles and exchange degeneracy

PROFESSOR: OK, so time for new subject. Let's introduce the subject and pose the questions that we're going to try to answer. And I feel that with identical particles, there's lots to think about, and it makes it into an interesting way to conclude the course. So identical particles.

So there is the issue of defining what do you mean by identical particles? And then the issue of treating them. So when do we say that two particles are identical? We say two particles are identical if all their intrinsic properties-- like mass, spin, charge, magnetic moment-- if all these things are the same, these two particles-- we have particle 1 and particle 2-- are said to be identical.

For example, all electrons are said to be identical. And if you think about it, well, what does that mean? You can have an electron moving with some velocity and an electron standing here, and they don't look identical. They have different states.

Well, they're identical in the sense that-- what we said-- the intrinsic properties are the same. Those two particle have the same mass. They have the same spin-- in principle. They have the same charge. Have the same magnetic moment. Have all these same properties.

Now, they can be in different states. One electron can be one momentum state, an electron can be in another momentum state. One electron can be in a spin state-- spin up. This can mean spin down. But what we would mean is that if you would put, by saying these particles are identical, we also mean they're indistinguishable.

What that would mean is that if one of you gives me an electron with spin up with some momentum state, and-- yeah, let's say spin up on some momentum state. And another one of you gives me another electron with same spin up, same momentum state, and I have those two electrons and I play with them for a minute and I give them back to you, you have no way of telling you got the same electron or you got your friend's electron. There's no possible experiment that can tell which electron you got.

So when we have identical particles like electrons-- elementary particles-- we understand what it means to be identical. Doesn't mean they're in the same state. It means that this is a particle that all the properties intrinsic of them are the same.

If you have a more complicated particle, you still can use the concept of identical particles. So, for example, you have a proton. A proton is a more complicated particle. It is made of quarks. And if you have two protons, they are identical in that same sense.

All the properties we can give to the proton-- the spin state of the proton, mass of the proton, the dipole moment of a proton, the magnetic moment of a-- all those are the same. If you prepare those protons in identical states, I cannot tell which is proton 1 and which is proton 2.

Then you have the neutrons. Neutrons are the same thing. The neutrons can be in several states, but two neutrons are considered to be identical.

We can complicate matters more. We can take hydrogen atoms. Are hydrogen atoms identical particles? And in quantum mechanics, we can think of them as identical particles-- or identical atoms, or identical molecules, or if you write the wave function for a hydrogen atom-- a new kind of entity of particle-- we will use the axioms of identical particles even for the hydrogen atom.

But you could say, oh, no, but they're not identical. A hydrogen atom can be in the ground state, or it can be in an excited state. But that's the same as saying this electrode is going with little momentum, and this is with high momentum. These are states of the hydrogen atom. Just like an electron has spin up and spin down, hydrogen atom has this state, that state, that state. If you arrange them in the same state, you cannot tell they are different.

So however clear these comments can seem-- or confusing, perhaps-- things can be a little subtle. In many ways, for example, physicists used to think of protons and neutrons as the same particle. Said what?

Well, that's the way they thought about it. It's a very nice thing. If you're working with energy scales, the proton and neutron mass difference is not that big, first of all.

So at some scale for the resolution of some experiments, or physicists that didn't have that many tools, the proton and the neutron were almost identical, and people invented this term called isospin. And you might have heard of it. It's a very famous symmetry of the strong interactions.

In fact, for the strong interactions, you have a nucleus. Whether you're a proton or a neutron doesn't make that much difference. So people used to think of this thing as an isospin state. Just the spin one half-- you have spin up, spin down.

Isospin means spin in some new direction that is unimaginable. But the isospin up would be the proton, the isospin down would be the neutron. And you will have a doublet. So people used to think of these two particles as different states of a nucleon, and then they would say, all nucleons are identical.

Why do you complain? A proton is the same as a neutron. It's just a different state of the isospin, just like the spin electron up or spin down is the same. So the power of the [? formalism ?] in quantum mechanics is that it allows you to treat these things as identical particles, and this makes sense. For you-- for your experiments-- these are identical things that you can think of different states of that. You may as well treat them that way.

So this is basically what happens. Now, so we define the identical-- I didn't write anything here. I'm going to get notes out today on scattering, and some of these things as well.

So in classical mechanics-- classical mechanics-- identical particles are distinguishable. And that's the main thing. How are they distinguishable?

Well, you have two particles, and I can follow-- whenever they're moving, I can say, OK, this is particle 1. This is particle 2. With quantum mechanics, you can do the same thing when they are really far away-- those particles-- and they don't come close together.

There's some sense in which classical mechanics sometimes applies, and that's when they're far away. Now, when the particles in quantum mechanics get close to each other, then you lose track which one is which. They occupy the same position. But in classical mechanics, they are distinguishable, because you can follow their trajectories, which is very nice.

You have an experiment. You follow the particle, say, oh, this is particle 1. This is particle 2. They're here together. They're going around. And they split, you follow the trajectories from the beginning to the end.

In quantum mechanics, there's no such thing as the trajectories. There's these waves. The waves mix together. They do things, then they separate out, and you just can't tell what they do.

There's another technique that we use in classical physics that it's probably also relevant. We can tag the particles. That means, if you're doing an experiment with billiard balls colliding, you could take a little marker and put a red dot on one of them and a black dot on the other one, and that tagging doesn't affect the collisions. And you can tell, at the end, where is the red ball, and which is the black ball.

We do the same with quantum mechanics. There's no way anyone has figured out the tagging a particle without changing drastically the way interactions happen. So it's a nice option in classical physics, but doesn't work in quantum mechanics.

Even in classical physics, we have something that survives. If you have a Hamiltonian for identical particles-- R2, P2-- that Hamiltonian is symmetric under the exchange. Whatever the formula is, it's not changed if you put r2, p2 and r1, p1.

It's a symmetric thing. The Hamiltonians have that symmetry, and there's no way to do this. So let's get to the bottom-- the real problem with identical particles with quantum mechanics.

We cannot tag them. Once these particles get together, you don't know what they did. You do an experiment of scattering in the classical mechanics, and you put two particles coming in, two detectors, and you tag the particles and you see what they do.

You do it in quantum mechanics, and you don't know if particle 1 did that and particle 2 did this, or if particle 1 did that and particle 2 did that. It's just not possible to tell. They're very different.

So how do we deal with this? Well, that's the subject of what we're going to do, but let's just conclude today by stating the problem. So the problem is that when we had distinguishable particles in quantum mechanics, we used that tensor product to describe a state. So for distinguishable particles, this thing which all particles, say, 1 up to n, we would use the tensor product and write PSI i1 for the particle one, PSI i2 for the particle two, PSI in for the particle n.

And this says particle one is in the state PSI i1, particle two is in the state PSI i2, PSI in. And these states are one of many states, for example. That's all good. And this is for distinguishable particles. And that's all correct.

Now, suppose you have two electrons, one up and one down. If they are indistinguishable, how are we supposed to write the state of the two electrons or to spin one-half particles, or maybe, in some cases, maybe some other particles. Am I supposed to write that the first particle is in state up and the second is in state down? Or are I supposed to write that the first particle is in state down and the second particle is in state up?

How do I describe the state with this one or with this one? They look equally plausible. So if you were in charge of inventing quantum mechanics, one possibility that it may occur to you is that if the particles are defined to be identical, then those two states should be identical. They should be indistinguishable, identical, physically equivalent. And this might be a good hypothesis to consider.

Unfortunately, that does not work. You say, why not? It seems so logical.

I cannot tell the difference between these two. Can I say that they're equivalent? Well, no.

If they would be equivalent, you could form a state PSI alpha beta, which is alpha times the first plus minus plus beta times the second minus plus. And I'm not going to write all the subscripts nor the tensors sometimes. With alpha and beta having this for normalization. That's a normalized state.

Now, if all those are equivalent, if those two are equivalent, all these are equivalent for all values in alpha and beta because the superposition of equivalence state is an equivalent state. But then let's ask for what is the probability that we find the two particles in the state PSI 0, which is plus along the x direction times plus along the x direction.

You know, whatever I do whether this hypothesis there that those two states are equivalent is correct, a state that is plus and plus can only be described one way-- plus plus. So I ask, what is the probability that this state PSI alpha beta in the plus along x and in the plus along x.

Now, you'll remember those pluses are states like plus, plus minus, 10 surplus plus minus with a 1 over square root of 2. And that becomes this. So this is 1/2 of plus plus plus plus minus, plus minus plus plus minus minus. That's the state.

So what is the probability that PSI alpha beta is found in the state PSI 0? It's this number. Now, if you do the inner product of these two vectors, only the mixes ones go with each other.

And this gives you 1/2 of alpha plus beta squared. And 1/2 of alpha plus beta squared is the inner product of these two states. And now you see that it depends on the values of alpha and beta because this is in fact 1/2 of alpha squared plus beta squared plus 2 real of alpha beta star.

And since alpha and beta are normalized, this is 1/2 plus real of alpha beta star. So this hypothesis that these two states are equivalent would mean that these states are equivalent for all alpha beta that are normalized. And then you would have that the probability to be found in PSI 0 would depend on what the values of alpha and beta you choose. So it's a contradiction.

So we cannot solve the problem of the degeneracy of identical particles by declaring that all the states are the same. So we have to find a different way to do it. And that's what we will do next time.