MITOCW | L19.1 Elastic scattering defined and assumptions

PROFESSOR: Today, we begin with our study of scattering. And so scattering is the next chapter in our study of approximations. What is the purpose of scattering?

Physicists want to study the interactions between particles, the forces. We know, for example, in a hydrogen atom there is an electromagnetic force within the proton and the electron. We understand it rather well.

But there are other forces in nature that we don't understand very well, are harder to understand. Those at higher energies, the interactions between particles can be rather complicated.

Also, it may happen that you have a situation where there's a potential affecting particles, maybe a trap or something that holds particles. And there's a potential. And you really cannot measure directly this potential. There's no simple way of doing so.

And then what you do is you send particles in. And you see how they get affected. And by studying the reflection or the scattering of the particles off of this potential, you'll learn a lot about what this potential can be.

So this subject today is scattering. And in general, you have a beam of particles, a target, and a number of detectors. That's what happens in a big accelerator. And in general, you have such situations. OK?

So a beam, some sort of target, and particles get scattered. And you have detectors. And in general, the detectors are all over in all directions because particles may back scatter, may go in different ways.

So collisions, in general, are rather intricate. Particles can change identity, can do all kinds of different things. For example, you can have a proton colliding with a proton. This could be the case when the beam is a beam of protons.

And here, maybe there's a target in which you study what happens when a proton encounters a proton. Or it may be that in some cases there's another beam that is coming also with protons. And they collide.

But when protons collide with protons, funny things can happen. For example, you can get the two protons plus a pion, a neutral pion, 0 denoting zero charge. That's a hadron, a strongly interacting particle, like the proton is. It's made of quarks.

And this can happen. There's no such thing as a conservation of the number of particles. That is one of the reasons you need quantum field theory to do this kind of computation.

And this reaction is allowed because it, at least the first thing you can check is that it conserves charges. Proton and proton go to proton and proton, so a charge is conserved. And pi 0 is neutral. So that, for example, can happen.

Or proton plus proton can go to proton plus neutron. Now, charge is not conserved. But this time, a positively charged pion called pi plus.

The particles can completely change identity. You can have an electron and a positron colliding. So that's electron. This is the positron, the anti-particle of the electron, equal mass, oppositely charged.

And then they can go into a mu plus plus a mu minus. These are charged laptons. Again, just like the electrical laptons, as opposed to these particles here. They're called hydrons.

These particles are, as far as we know, are elementary. These particles are made out of quarks. And in this case, you have a complete change of identity. The original particles have disappeared. And new particles have been created.

And you can look forward to study these processes as you continue your studies of quantum mechanics. But these are like reactions. Now, in our name, in our nomenclature for scattering, we will call the process a scattering process where the identity of the particles is unchanged.

So we will have scattering, strictly speaking, when there is no change of identity in the initial and final states. No change of identity of the particles in the process of going from initial to final state. So a scattering process looks like a plus b goes to a plus b. We don't change identity.

Those processes where you change identity are harder to discuss. And we won't discuss them here. Even here, there is still a lot of things that can happen. It's very intricate.

So we will demand even more. We will call or look for elastic scattering. And that means that if these particles or objects have internal states, those internal states are not changed. So the internal states of the particles do not change.

Of course, that can happen essentially when the particles are complex. They're bound states of other particles. So you may have a bound state of one form. And then when you have bound states, you know you have all kinds of energy levels internal states.

And then when we say we have an elastic process, it means that those internal states are not changed. A classic example of an inelastic scattering experiment, the most famous historically of those experiments, is an experiment by Frank and Hertz in 1914. They were having a chamber with mercury gas. And they shot electrons from one side to the other.

And the electrons and the mercury atoms collided. And they found that the electrons were slowed down by some quantized amount of energy. And that was the first evidence that atoms had energy levels. Bohr's theory of atoms had been proposed one year before that, 1913.

And then, the process in which these electrons hit the atoms and lost energy corresponding to producing transitions in the atoms. And that would be an inelastic scattering because the atom changed its internal state. This is a collision between an electron and an atom, but the atom has changed internal states.

So we will want to consider cases when we don't change the internal state. And that will be when we have elastic scattering.

So a few more things that we're going to assume as we do elastic scattering. We will work without spin. All we will do in the next lectures, one or two more lectures in this, will be particles without spin.

As we're doing in our course, we also work in the nonrelativistic approximation. So these are the first things that we will assume. One, no spin. This doesn't complicate matters as far as the scattering is concerned. It just complicates the algebra and the calculations you have to do because you have more degrees of freedom.

We will be working nonrelativistically. Moreover, we will assume there are interactions. There are interactions between the particles that produce the scattering.

And those interactions are simple enough that they just depend on the difference of position. So the interaction potentials are of the form v of R1 minus R2. So processes in which will have two incoming particles and they interact, and they scatter elastically. And the potential depends on just the difference of the two positions.

Eventually, we will even assume it's only on the magnitude of these differences for particular cases. But if you have a potential like this, like when we were studying with hydrogen atom for the first time, a potential that just depends on the differences of positions can be treated in the center of mass frame. It's a nice frame where you can work on it.

And then you can think of it as a single particle scattering of a potential. So you translate the Schrodinger equation into a center of mass degrees of freedom that are generally simple. And we don't worry about and a relative degree of freedom.

So this can be done here. So this makes a process equivalent to scattering of a particle of a potential V of r. And that particle has the reduced mass, just like happened with a hydrogen atom. The real mass of the equation we solve for showing the Hamiltonian for a hydrogen atom, the mass that shows up there is the reduced mass, which is approximately equal to the electron mass. But in general, in this scattering process, it may be between two identical particles in which case a reduced mass would be half the mass of the particle.

So again, we will work scattering with energy eigenstates. You may have studied already in 804 a little bit of problems of scattering off of rectangular barriers, tunneling, all these things. And we work with energy eigenstates in those cases. And we will work with energy eigenstates as well here.

Now, in some versions of 804, you discuss a lot wave packets. I used to discuss a lot wave packets. We would build wave packets and send them in. And use a transmission on reflection coefficients to figure out what the wave packets do. And in general, a wave packet is a somewhat more physical way of thinking of the processes. Because the energy states, eigenstates, are unnormalizable and don't have a direct interpretation in terms of particles.

Rigorously speaking, you should always do things with wave packets. But the fact is that we all work with energy eigenstates. And most of the times, what happens with wave packets can be more or less gleaned from what is happening with energy eigenstates.

So we will not new wave packets here. We will not bother to construct wave packets. They would not teach us too much at this moment. So we will work with energy eigenstates.