PROFESSOR: OK, time for us to do one example, a non-trivial example, which is the ionization of hydrogen. It's a fun example, and let's see how it goes.

So ionization of hydrogen. Ionization of hydrogen. Very good. So we're going to think of a hydrogen atom on its ground state sitting there, and then you shine an electromagnetic field, and if the electromagnetic field is sufficiently strong, the electron is going to be kicked out by interacting, typically with the electric field of the wave.

So the wave comes in and interacts. So we usually think of this in terms of photons. So we'll have a hydrogen atom in its ground state. And then you have a harmonic e field, and the electron could get ejected. So typically, you have a photon in this incident.

We think of this in terms of electromagnetism, although our treatment of electromagnetism was not going to include photons in our description. But we physically think of a photon that this incident on this-- here's a proton. Here is the electron going in circles, and the photon comes in, a photon gamma, and it kicks the electron out.

So the photon has energy e of the photon is h bar omega. So when we think of putting a light beam, and we're going to send many photos, we have to think of each photon, what it's doing-- whether the energy is very big, whether the energy is very small, and how does it affect our approximation.

So half of the story of doing such a calculation is understanding when it could be valid, because we're going to assume a series of things in doing the calculation. And the validity still won't turn out to be for a rather wide range of values. But we have to think about it.

So a couple of quantities you'll want to consider is that the energy of the electron, energy of the electron that is ejected is h bar squared k of the electron that is ejected, k squared over 2m of the electron. And it's going to be equal to the energy of the photon minus the ground state energy of the electron in the hydrogen atom.

So the hydrogen atom here is energy equal 0. Ground state is below 0. So if you supply some energy, the first part of the energy has to be to get it to energy 0, and then to supply kinetic energy. So the kinetic energy is the energy of the photon minus what this called the Rydberg, or Ry. The Rydberg is 13.6 eV with a plus sign. And that's the magnitude of the depth of the

well.

The Rydberg is e squared over 2a0 or 2 times the Rydberg is e squared over a0. That's what was calculated, and this is actually equal to h bar c the constant alpha times h bar c over a0. remember, the constant alpha was e squared over h bar c. So those are some quantities. OK.

So we need the electron to be able to go out. The energy of the photon must be bigger than a Rydberg. OK, so conditions for our approximation. One. We're going to be using our harmonic variation. We said in our harmonic variation, Hamiltonian delta h was 2h prime cosine of omega t, and we want this h prime to be simple enough. We want to think of this photon that is coming into the atom as a plane wave, something that doesn't have big spatial dependence in the atom. It hits the whole atom as with a uniform electric field. The electric field is changing in time. It's going up and down, but it's the same everywhere in the atom.

So for that, we need that-- if you have a wave, it has a wavelength, if you have an atom that is this big, you would have the different parts of the atom are experiencing different values of the electric field at the same time.

On the other hand, if the wavelength is very big, the atom is experiencing the same spatially independent electric field at every instant of time. It's just varying up and down, but everywhere in the atom is all the same. So what we want for this is that lambda of the photon be much greater than a0. So that-- 1.

So this means that the photon has to have sufficiently long wave, and you cannot be too energetic. If you're too energetic, the photon wave length is too little. By the time it becomes smaller than a0, your approximation is not going to be good enough. You're going to have to include the spatial dependence of the wave everywhere. It's going to make it much harder.

So we want to see what that means, and in the interest of time, I will tell you with a little bit of manipulation, this shows that h omega over a Rydberg must be much smaller than 4 pi over alpha, which is about 1,772. So that that's a condition for the energy. The energy of the photon cannot exceed that much.

And let's write it here. So that's a good exercise for you to do it. You can see it also in the notes just manipulating the quantities. And this actually says h omega is much less than 23 keV.

OK, 23 keV is roughly the energy of a photon whose wavelength is a0. That's a nice thing to

know. OK. While this photon cannot be too energetic, it has to be somewhat energetic as well, because it has to kick out the electron. So at least, must be more energy than a Rydberg.

But if it just has a Rydberg energy, it's just basically going to take the electron out to 0 energy, and then you're going to have a problem. People in the hydrogen atom compute the bound state spectrum, and they computer the continuous spectrum in the hydrogen atom, in which you calculate the plane waves of the hydrogen atom, how they look.

They're not all that simple, because they're affected by the hydrogen atom. They're very interesting complicated solutions for approximate plane waves in the presence of the hydrogen atom. And we don't want to get into that. That's complicated. We want to consider cases where the electron, once it escapes the proton, it's basically a plane wave. So that requires that the energy of the photon is not just a little bit bigger than a Rydberg, but it's much bigger a Rydberg.

And saw the electron, the free electron does not feel the Coulomb field of the proton. And it's really a plane wave. So here, it's a point where you decide, and let's be conventional and say that 1 is much less than 10. That's what we mean by much less 1/10.

So with this approximation, h omega must be much bigger than 13.6. So it should be bigger than 140 eV. That's 10 times that. And it should be smaller, much smaller and 23 keV, so that's 2.3 keV. So this is a range, and we can expect our answer to make sense.

If you want to do better, you have to work harder. You can do better. People have done this calculation better and better. But you have to work much harder. I want to emphasize one more thing that is maybe I can leave you this an exercise. So whenever you have a photon in this range, you can calculate the k of the electrode, and you can calculate how does the k of the electron behave. And you find that k of the electron times a0 is in between 13 and 3 for these numbers.

When the energy of the photon is between those values, you can calculate the momentum of the electron, k of the electron. I sometimes put an e to remind us of the electron. But I'll erase it. I think it's not necessary. And that's the range.

OK. So we're preparing the grounds. You see, this is our typical additive. We're given a problem, a complicated problem, and we take our time to get started. We just think when will it be valid, what can we do. And don't rush too much. That's not the attitude in an exam, but

when you're thinking about the problem in general, yes, it is the best attitude.

So let's describe what the electric field is going to do. That's the place where we connect now to an electric field that is going to produce the ionization. So remember the perturbation of the Hamiltonian, now, it's going to be the coupling of the system to an electric field. And this system is our electrons. So it's minus the electron times the potential, electric potential, scalar potential.

Now, needless to say, actually, the electron that is going to be kicked out it's going to be non-relativistic. That's also kind of obvious here. You see h omega is 2.3 keV. You subtract 13.6 eV. Doesn't make any difference. So that's the energy of the emitted electron roughly, and for that energy, that's much smaller than 511 keV, which is the rest mass of an electron. So that electron is going to be non-relativistic, which is important, too, because we're not trying to do Dirac equation now.

So here is our potential, and then we'll write the electric field. Let's see. The electric field. We will align it to the z-axis to begin with. 2e0 cosine omega t times z hat. These are conventions. You see, we align it along the z-axis, and we say it has a harmonic dependence. That's the frequency. That's the frequency of the photons that we've been talking about, and the intensity, again, for these convention preference will put 2e0 times cosine omega t.

So some people say ep, which is the peak e field is 2e0. That's our convention. Well, when you have an electric field like that, the electric field is minus the gradient of phi, and therefore, we can take phi to be equal to minus the electric field as a function of time, times z.

So if you take the gradient minus the gradient, you get the electric field. And therefore, we have to plug it all in here. So this is plus e, e of t, z, and this is e. e of t has been given 2e0 cosine omega t. And z, we can write this r cosine theta. In the usual description, we have the z-axis here, r, theta, and the electric field is going in the z direction. So z's are cosine theta.

So I think I have all my constants there. So let's put it 2 e, e0, r cos theta cos omega t, and this is our perturbation that we said it's 2h prime cosine omega t. That's harmonic perturbations were defined that way. So we read the value of h prime as this one. e e0 r cosine omega t. No, r cosine theta.

OK. We have our h prime. So we have the conditions of validity. We have our h prime. Two more things so that we can really get started. What is our initial state? The initial state is the

wave function of the electron, which is 1 over square root of pi a0 cubed e to the minus r over a0. That's our initial state.

What is our final state, our momentum eigenstates of the electron? So you could call them psi, or u sub k of the electron. And it would be 1 over l to the 3/2, e to the ik of the electron times x.

These are our initial and final states. Remember, the plane waves had to be normalized in a box. So the box is back. u is the wave function of the plane wave electron. And we could use a plane wave, because the electron is energetic enough. And it has the box thing. This is perfectly well normalized. If you square it, the exponential vanishes, because it's a pure phase, and you get 1 over l cubed. The box has volume l cubed. It's perfectly normalized. This is all ready now for our computation.