PROFESSOR: How do we think of the Born approximation?

Well, we can imagine a diagrammatic expression when we use propagators. Propagators are things that take a signal and propagate it in some direction. So think of your scattering center here, and, of course, we were supposed to look far away but I don't have enough room on the blackboard. I'll take this point to be far away.

And now what does it say, this equation? It says that the first Born approximation, you get that r, the incident wave. The incident wave has reached r. This is the incident wave of r. So I will represent this as a wave that reaches the point r. That's the first term on the right hand side, a line reaching r. Because a wave has come in that direction and reached r.

On the other hand, this is a little different. What is the physical interpretation of this term? This is a wave that reached r prime. And this wave was shaken by the potential and created the source, as if the potential of r prime was a medium. The wave reached r prime, shook the potential, and then a wave appeared that was propagated by the propagator, the Green's function. It took it from r prime, it took it to r.

So our interpretation of this diagrammatically is that another part of the incoming wave came here and got to the point r prime. At the point r prime, it interacted with the potential, which is a medium, and out came a wave in the direction of r and reached there. And r prime is supposed to be integrated. But that's kind of in the figure.

So this is the zeroth order term, this is the first order term, wave that reaches r prime, wave that reaches [INAUDIBLE]. Now I can include the second term. I could include it in this diagram but I would confuse things. So let's draw another one. [INAUDIBLE] thing doesn't look the same. I won't get them to look the same. Doesn't matter.

Here is r still, and what is our second wave doing? Our second wave, we could put it here and imagine that you have here the e to the ikr. So here you would have e to the ik dot r double prime. So the wave reaches the point r double prime, which is here. And from the point r double prime, it interacts with the medium-- that's that heavy dot-- and gets propagated into r prime, which is also inside the material. So another wave, and here is r prime. And once it's propagated into r prime, it interacts with the material u-- that's the heavy dot-- and gets propagated into r.

So that's the second order Feynman diagram. The zeroth order first, Feynman diagram, the wave just reaches. First order, the wave comes, interacts once with the potential, and re-emits a wave over here. Second order comes here, interacts with the potential, emits a wave, interacts at another point, emits another wave, and gets to you. That's a pictorial description of a Born approximation, which makes it intuitive, makes it clear.

It is like Feynman diagrams of field theory, in which you have some set of initial state, some set of final state, and you sort of find the ways the particles arrange themselves. If you have an electron and a photon scattering, you have an electron here and a photon scattering, and then photon and electron out scattering. What do they do? And then Feynman method, you think, OK, they do all what they can possibly do. They get the photon, interacts with electron, electron propagates and re-emits a photon, or it may be that the photon gets emitted first and then the incoming photon hits the other place and it goes that way.

All these things can happen. And here you have all the ways in which the waves can scatter and re-scatter back to reach the observer. Basically, you're finding all possible ways the waves interact with the medium and reach you. And that's a very intuitive way. You think of it, there the material, you're there, what do I get? Well, you get the waves that just hit you directly. And then you get the waves that hit the material and scatter to you. And then you get the waves that hit the material, scattered, hit the material again, and then got to you. And then you get the waves that did that several times. A very nice, simple picture.

So that is the Born approximation. We can do one example. Let's do one example. And assume, simplify this formula. So we did this formula. And if the potential is not spherically symmetric, you can't do more with it. But if the potential is spherically symmetric, that first Born formula, which is a very famous result, admits simplification and you can do some of the integral. You can do the angular part of the integral because the potential is spherically symmetric, and you know how things depend on angle.

So it's a nice thing to have a boxed formula that already does for you this calculation when the potential is spherically symmetric. So if v of r is equal to v of r, which is spherically symmetric, we go back to that formula and we would have f of k of theta, we claim, we'll see if that is true, minus 1 over 4 pi. Now this u there, remember, this u was just the scaling of v in order to make the Schrodinger equation look simple. So that involves a factor of 2 m over h bar squared. You still have the cube r prime, e to the minus i k dot r prime, v of r prime.

So what is the intuition of such formulas? I think it's important for you, maybe you haven't seen these formulas before, if you look at the formula like this. What do you expect the answer to depend on? What is that integral going to depend on? Any opinion?

Well, it certainly will depend on the potential, but now if it's very symmetric, I might as well delete that. Is that right? We said it's spherically symmetric. So it will depend on the potential. That's, of course, true. But will it depend on the vector k? Yes, it better depend on the vector k. But all of the vector k or some of the vector k?

It's good to think about that before doing the integral.

Well, the interesting thing is when you have an integral in this form in which this doesn't depend on the direction of r, this is our prime, but in fact, you see, there's nowhere in this formula anything to do with r and r prime. This is an integral all over space. So in fact, I can also delete now the primes, integrate them over all of space. There's no r on the left hand side, there's no r left, so what for carrying primes? No need for that.

Now what I want to explain here is that this integral can only depend on the magnitude of the vector k. Cannot depend on the direction of the vector k. And because it depends on the magnitude of the vector k, it only depends on theta, which is what we wanted to say. So if before, you write

For theta, you could have asked, well, how do you know? This interval can depend on phi, k dependent on phi, but no, this integral doesn't depend on the direction of k. And it doesn't because you can choose your axis to do this integral whichever way you want. This is a dummy variable of integration. So if vector K points in this direction, you could choose the z-axis to go there, and that's it. It's only going to depend on the magnitude of that vector. So that's an important concept in here.

Oh, and you can think it another way. This answer, if I would have put another vector k prime here that is obtained by rotation, think of k prime, I put here k prime, which is obtained by a rotation, acted by a rotation matrix on the vector, say, from the left. Well, you would say, OK, if it's acted by a rotation, I could instead make the rotation act on the vector r. This potential doesn't change on the rotations and this measure is invariant on the rotation. So at the end of the day, this integral does not depend on the rotation you did.

So bottom line, this integral just doesn't depend on the direction of k, and we can do it by putting k-- or saying k is here and now making it explicit by putting r prime here and calling this angle theta. And we just do it.

So if we call it that way, this integral is, you have minus m over 2 pi h squared, and you have the volume integral, which is 2 pi for the phi integral. So we'll have a theta and a phi that you integrate with respect to k, and the phi integral will always give you 2 pi and will leave the theta integral here. So 2 pi. And the r integral is r squared dr. The theta integral in spherical coordinates is v cos theta. And you have here e to the minus i length of this, length of that, cosine theta times v of r.

So things simplify even more. So the 2 pis can [INAUDIBLE] minus m over h squared. The radial integral, it's 0 to infinity, r squared vr, v of r, and you have the angular integral, and that angular integral is easily doable. It's that and this. You can [INAUDIBLE] cosine theta u and just do the integral. That integral gives you 2 sin of Kr over kr.

And our result, therefore, is, you shouldn't confuse the capital K with the lowercase k, which is the magnitude of the momentum or the magnitude of the energy. So we're done, basically. And what do we have for fk of theta? We have minus 2m over K h squared, integral from 0 to infinity of r, dr, one r got cancelled, the k went out, r dr, v of r, sine of Kr.

So the Fourier transform got partially done. And here, K, as you remember, is 2 k sine theta over 2.

So there we get the Born approximation simplified as much as we could. We couldn't do much better anymore without knowing what the potential is. But once you know the potential, it has become just the simple integral. So while the concepts that led to this result are interesting, and they involve nice approximations, by the time you have this result, this is very useful. You can put an arbitrary potential v of r, calculate the integral, and out you get the scattering amplitude.

So this is done, for example, for Yukawa potential, v of r minus 1 over r-- well, we will put the beta-- e to the minus mu r. That's called a Yukawa potential.

It's the kind of modification of the electromagnetic potential in the limit in which the photon would acquire a mass. That was sort of the intuition of Yukawa. If you see-- this has units of [INAUDIBLE] physically mass and h bars and things like that. But if you let this thing go to 0, if mu goes to zero, you recover coulomb potential.

So that's the relevance of the Yukawa potential. Yukawa invented it while thinking of pions and thinking of the forces transmitted by pions, who are light hadrons and strongly interacting particles, but with low mass, relatively low mass compared to a proton, one fifth of it or even less, maybe, one seventh, one eighth.

And then you would have a potential of this for and you can study scattering when you have this kind of formula. So this is solved in many books. It's also in the textbook, and in this case, f of k, the integral can be done exactly. You can plug v of r there. And the answer is that fk of theta is equal to minus 2 m beta over h squared mu squared plus K squared.

And K squared would be this quantity. So there is a theta dependence, and you can look at this formula, compare with the Coulomb k, [? take ?] limits, plot it, it's kind of interesting.

So it's easily done, and you can play with many potentials. All right, so we're done with scattering. Done like three lectures and a half on this subject, and I'm going to now turn into a new subject, our last subject, which is we're going to try to explain some of the basics of identical particles.