PROFESSOR: Good morning. Today's lecture will deal with Zeeman effect. And then we'll get started with a semi-classical approximation. So Zeeman in effect is the last topic we do with respect to the hydrogen atom and the corrections of perturbation theory and with WKB or the semi-classical approximation, we begin a new chapter in 806.

Zeeman effect. So this is an effect having to do with an atom in a magnetic field. It was discovered by a Dutch physicist, Peter Zeeman, who lived from 1865 to 1943, who was a Dutch. The work was actually done in 1896 at a time where there was very little idea of quantum mechanics to be, and he got a Nobel Prize for in the year 1902.

So what he discovered was the spectral lines seemed to split in the presence of a magnetic field. It's an old result, therefore, 100 years old. Its explanation and understanding took about a couple of decades, because you couldn't do it without quantum mechanics. So nobody could quite figure out what had happened, but was very, very important in its time. It still remains very important. People use the Zeeman effect all the time.

In fact, it's used nowadays in studies of astrophysics, studies of the sun, the sunspots. You know, there are places in the sun, where the temperature is a little lower, and that's places where the magnetic field lines in the sun sort of breakout from the interior to the exterior. And it's interesting, because the sun produces this cycle of solar spots, and it happens, because the sun doesn't rotate uniformly, has a different rotation speed in the equator, faster than in the poles, so the magnetic field lines that go from north to south in the sun get tangled up, and they start breaking up and doing all these things.

So people want to know what is the magnetic field in the solar sunspot. And in fact, they observe the weak magnetic fields away from the solar sunspots, and then they see the spectrum of an atom. And suddenly, as you move inside the spot, the field, the spectrum, the lines split. And they can measure the magnetic fields very accurately. They're of the order of 3,000 Gauss, 2,000, 3,000 Gauss.

And it's pretty interesting work. So this Zeeman effect remains very important. So what is the Zeeman effect? We have Zeeman effect here. We have the magnetic field interacting with the electron, and the electron now has two magnetic moments, a magnetic moment associated with the orbital motion.

This looks completely like the classical formula of the magnetic moment due to a particle that goes in circles. It produces a current, and that current is proportional to the angular momentum of the rotating particle. And of course, there is the magnetic moment due to the spin that has a factor of 2 here. This g factor, we've discussed before. S.

So H Zeeman, you put an external magnetic field into the atom, a constant uniform external magnetic field, and you have a minus mu dot b, so you have a interaction mu l plus mu s, dot B. So this is e over 2mc. That's the typical factor here, and a recognizable l plus 2s. So it's not l plus s. It's l plus 2s times b.

And many times, we think of B, align the axis so that it is in the z direction. So this turns out to be e over 2mc lz plus 2sz times B. So this is the Zeeman Hamiltonian. But this is part of a story of an atom. So if we want to think of the hydrogen atom properly, we must consider and reconsider what was the Hamiltonian there. And we had an H for the hydrogen atom. That was an H 0.

That was the familiar one, p squared over 2m minus e squared over r. Then we had a fine structure Hamiltonian, delta H, fine structure. Let's put fs for fine structure. Those were the relativistic terms, the Darwin term, and the spin orbit coupling. The three of them constituted what we call defined structure Hamiltonian. And now we have a Zeeman effect. I probably should go delta H Zeeman, because it's an addition here to the term we had before. It refers to what I call just H Zeeman. But in the context of the hydrogen atom, we should call it delta H Zeeman.

And now, we have to rethink. And the reason the Zeeman effect is non-trivial for us, and it's a very interesting and somewhat challenging example of what we have to do in perturbation theory, is that we cannot forget about the fine structure. So if it would be just this, it would be kind of simple. But we have the whole thing.

So we have to make an approximation sometimes. And we're going to consider two interesting cases, the very weak Zeeman effect and the very strong Zeeman effect. I will not consider the intermediate Zeeman effect, not because it's not interesting, but because there's very little you can do to simplify it, and to think about it. You basically have to go ahead and diagonalize the large matrix.

So while it's important, and if your life depended on this, and your research dependent on this, you would do it. For us, we have a lot to learn from the weak case and the strong case, lots of concepts, and we'll leave the intermediate one for later. So how can we decide how to treat these terms?

Well, we should look physically at what's happening. We have a magnetic field, an external magnetic field. But the fine structure constant taught you that there is something like an internal magnetic field in the atom. It's that magnetic field responsible for spin orbit coupling is that magnetic field that the electron sees when it's going around the proton. There's a static electric field, but whenever you are in motion, a static electric field in the lab also has a magnetic field by relativity, and you do see a magnetic field. You could also imagine it as you are the electron, and the problem is going around you, and creates at the center of the loop, a magnetic field.

In any case, you've looked at that magnetic field. And we can call it the internal magnetic field due to spin orbit. As one exercise in the homework, you've been asked to estimate the value of that internal magnetic field. Is it a Gauss, 1,000 Gauss, 10,000 Gauss? How much it is for a typical level?

So there's a number here that this interest. And now we can decide whether we have a weak Zeeman effect or a strong Zeeman effect, by looking how your external magnetic field compares with this little magnetic field. So we'll have a weak Zeeman effect, A. Zeeman. When B is much smaller than B internal.

And therefore, the effects of Zeeman is going to be smaller than fine structure. So really, when you look at this line, and you have the first two terms, you should think of these two terms as your new known Hamiltonian. And the Zeeman term, at its perturbation.

Yes. You've solved for the fine structure coupling and the shift. So this is the known thing. These are going to be the known states with known energies. You only know them the first order, but you know them. And then the Zeeman effect will be a perturbation theory on this.

This is the weak Zeeman effect. How about the strong Zeeman effect? So B will be strong Zeeman. This time, the magnetic field associated with the Zeeman effect, the external magnetic field, is much smaller than the magnetic field responsible for spin orbit coupling.

And this time, what are we going to do? Well, we will take the Hamiltonian. It will be H 0. The strong Zeeman effect means this Zeeman Hamiltonian is a lot more important than the spin orbit coupling. So we'll add delta H Zeeman here. And we hope this will be our known Hamiltonian, because anyway, the Zeeman stuff is much stronger now than fine structure. So this should be our non-Hamiltonian. You should complain, no, this is not known. I haven't done Zeeman, but we'll look at it. And once we have our known Hamiltonian here, spin orbit has to be rethought. Fine structure has to be rethought as a perturbation.