

**PROFESSOR:** So B strong field Zeeman. So what did we say we would do? We would have  $H_0$  plus  $e$  over  $2mc B L_z$  plus  $2L_z B$ . This would be our  $H_0$  check I call it, plus  $\Delta H$  fine structure 1.

And we said what we have to do now is strong field Zeeman is more important than fine structure. So we first have to get the strong field Zeeman figured out, with the bare bones hydrogen atom, what it does. And then on those states, we will do perturbation theory for fine splitting. So it is redoing fine splitting.

And you say oh my god, that's hard. We spent a whole lecture doing that. Well, second time you do things, they go a little faster. So it's not that bad.

But here there is something quite remarkable. This was supposed to be your known Hamiltonian. And you say no, it's not known. I never solved this before. On the other hand, when we had  $H_0$  plus  $\Delta H_1$ , a fine structure, we did struggle. And we found those states-- approximate states.

Here the situation happily is surprisingly simple. And one reason for that is the following. That again, perhaps your initial impression this can be solved exactly. You don't even need perturbation theory to add this term. What? Yes!

This Hamiltonian commutes with  $H_0$ . Isn't that right?  $H_0$  is rotational invariant. So it commutes with any  $J$ . Uncertain  $H_0$  has nothing for a spin.

It's a one matrix there. So this commutes with Hamiltonian. So it's possible that you can diagonalize this completely. Simultaneous-- eigenstates of the first part and the second part, so simultaneous eigenstates from all of those! But the news is even better.

Your uncoupled states-- uncoupled states  $n, l, m, l, m, s$ , those were eigenstates except eigenstates of  $H_0$ , the all good ole hydrogen atom. But actually they are exact eigenstates of  $L_z$ , and exact eigenstates of  $s_z$ . So they're exact eigenstate of the Zeeman Hamiltonian. So they state are it. These are the exact states of  $H_0$  hat!

These are exact eigenstates-- eigenstates of  $H_0$  check-- I'm sorry. Its was not hat with eigenenergies as follows. There are no mystery, the eigenenergies, they're very simple. The eigenenergies are  $e, n, l, ml, l, m, s$ . Exact are  $e, m, 0$ , the ones that the hydrogen atom has that don't depend on any of these other things plus  $eB$  over  $2mc$   $\hbar$   $B$  over  $2mc$   $ml$  plus

2ms.

So this serves perfectly the name of known Hamiltonian. That was not the case for weak Zeeman. And weak Zeeman who had this one and this one, and the other was the approximately known Hamiltonian, to which we added the weak Zeeman. Here, it's this perfectly known Hamiltonian, to which we have to now add fine structure.

So maybe the last thing that helps you visualize what's going on is to understand what happens to the splittings. Because you're going to have to fine structure splitting. And fine structure again, you will have to ask, can I use non-degenerate perturbation theory or can I not use it? So you need to know what happened with the degeneracies after you add this term. Are all that the degeneracies of the hydrogen atom broken by this term, or do some survive?

If they survive, is fine splitting diagonal in those degenerate subspaces or not? So the most important thing is figuring out what are the degenerate spaces after you've added this term. This is intuitively what you have to do. If you approach this problem OK, I now have to compute fine structure on a new basis, and you have no idea what the new basis is, you are proceeding a little bit with your eyes covered. You should always try to make things a little more concrete.

So look at the  $n = 2$  states. You have  $l = 0$  and  $l = 1$ . Here, you have six states. Remember, their spin. There's two states here.

The two states of  $l = 0$  have  $m_s = 1/2$  and  $m_s = -1/2$ . So they're going to split. This number is going to be either plus 1 or minus 1, and they're going to split. And here-- so this is plus 1 on this factor here, this number, or minus 1 for that number.

For this states of  $l = 1$ ,  $m_l$  for example, can be 1, and  $m_s = 1/2$ -- so 1 plus 1 is 2. So there is a state of 2.  $m_l = -1$  and  $m_s = 1/2$  gives you 0. So that's another state.

You can have more states. For example, if you take  $m_l = 1$  and  $m_s = 2 - 1/2$ , you get the 0. But you can also have  $m_l = -1$  and  $m_s = 1/2 + 1/2$ , which also gives you 0. So here there's a degeneracy.

There are six states. You will see that there is one here and one there. So here is the nature

of the degeneracy. The six states have split like that. There's a degeneracy across  $l$  multiplets.

And there's a degeneracy within  $l$  multiplets. So two types of degeneracy. And that's what you will have to consider when you think of the fine splitting. In fact, the problem in the homework gives you some sort of trickery to evaluate this expectation value with a little less work than the traditional method, but still asks the question whether you can use the generator or non-degenerate perturbation theory. And that's an interesting question.

And this example, can help you visualize a little better what kind of degeneracies you have.

OK, we're concluding a chapter in the history of [INAUDIBLE] six. We're done with perturbation theory, and we're done with hydrogen atom for the moment.