PROFESSOR: It's not unsuspected, because in some sense, energy eigenvalues, what you need, the first order energy's then split the state. The hope is that the second order energy split the states, so that's why you have to go to second order. So going to second order is something we had to do. There was no hope to do this before we go to second order.

So let's go to second order, and order lambda squared. And we put n0l state in.

The left hand side, happily, is 0. So we have n0l from the right hand side, en1 minus delta h psi 1. But again, psi 1 has two pieces, a piece in the space we had that we just calculated plus a piece in the space of v tilde n and 1 minus delta h psi 1 vn plus the energy, which would be en2 in here. And we hit with n0l, so we pick an al0.

OK, that doesn't look that terrible. I don't know if you agree, but it really doesn't. Especially because a few things are gone. This term is zero. Why? Because state in the degenerate subspace orthogonal or to v hat.

Here I want to remind you of what we did. We did a long computation to explain that even though delta h is only diagonal in the degenerate subspace, when you have a state here in the degenerate subspace, you can let delta h think of it as acting as an eigenvalue. But that's a great thing, because if this acts as an eigenvalue, this is the first order of correction. But all the first order of corrections are the same, so this here, delta h and this, is going to give the same as en1 on this state, and therefore this whole state, happily, is all 0.

So it's again, this delta h-- we did it with a resolution of the identity. Hope you remember that argument. If you don't, look at it back later. But with a resolution of the identity, we argued that delta h, when acting in a state of vn on the right, you can assume that this is an eigenstate of it.

So this whole term is zero. So now we are in pretty good shape, in fact. The equation is not that bad. The equation has become minus n0l, delta h, psi 1, on v hat plus en2 al0 equals 0.

And this psi 1, we've already calculated it there. So this is great. You see, you should realize that at this moment we've solve the problem, because we're going to get from here something complicated acting on a 0. Because psi 1 has this a1's, but the a1's were given in terms of a0. So something on a 0, something on a 0, it's going to be an eigenvalue equation for a0, an

eigenvector equation for a0.

So let me just finish it.

So I have to do a little bit of algebra with this left hand side. I can put here this ap1 times the p0 states there. So on the left it will be minus the sum over p, n0l, delta h, p0 times ap1 plus en2 al0, 0. OK, now I just have to copy that thing there. And I better copy it, because we really need to see the final result. It's not that messy.

So here it is. I'll copy this thing. I also can write this as delta h nlp. You recognize that thing. So this will be the sum over p of delta h nlp. The sine, I will take care of it, times that thing over there, 1 over ep0 minus en0. That's another minus sign that cancels this sign here, and I have here the sum over k equals 1 to n, delta h, pmk, ak0 plus en2 al0 equals 0.

OK, so what do we do now? Just rewrite it one more time and it will all be clear. So I'll write the k outside, k big parentheses, p. For the first term I'm going to pull the k sum out and we'll sum over p first. So what is it? Delta h nlp delta h pmk. Those were the two things here.

And then we have the denominator. I'll change a sign of the first term. I will change its sign so that the second term looks more like an eigenvalue. So I changed sign here by changing the order of things in the denominator, and then I put minus en2. I don't have a sum over k here, but we can produce one by writing delta kl ak0 equals 0.

Look, we got our answer. If I call-- invent a matrix mlk2, which is precisely delta h nlp delta h pmk over en minus ep is sum over p. This is a-- you see, you sum over p, n is irrelevant, you have a matrix here in lmk. That's why we call it mlk. And therefore this whole equation is the sum from k equals 1 to n of mlk2 minus en2 ak0-- ml2 en2 delta lk ak0 equals 0. Or, if you wish, it's just a matrix equation of the form m2 minus en times the identity on a vector equals 0.

And this is an eigenvalue equation. So finally, here is the answer. Let's say you compute this matrix of order lambda squared with this perturbation and that is the matrix that if you diagonalize, you find the second order energy corrections and you find the eigenvectors. And if you found the eigenvectors, you found the good basis.

So the problem has been solved at this stage. You now know that if you fail to diagonalize, to break the perturbation of the first order, you will have to diagonalize a second order matrix.

And once you diagonalize it, you now have the good basis, and it will happen that if all the levels get split at second order, we can calculate fairly easily the rest of the pieces of the [? states. ?]

So we'll leave it. There you'll complete some details, more elaborations of that in the exercises, and we'll go to hydrogen atom next.