PROFESSOR: With this, we can phase the construction of the operators that are going to help us build totally symmetric states and totally anti-symmetric states, and understand why we solve the problem of degeneracy, exchange degeneracy. So let us look into that.

So this is called complete symmetrizers and anti-symmetrizers, complete symmetrizers and anti-symmetrizers. Permutation operators don't commute. So there's no hope ever of simultaneously diagonalizing them. There's some of them are not even Hermitian. So even worse, you cannot find that, oh, I'm going to get a complete basis of states, simultaneously diagonalizing, some permutation operators are Hermitian. The majority are a unitary. The transposition operators are the Hermitian ones. Those you could try to diagonalize. But if you have the whole permutation group, you cannot diagonalize it. It's just too many things that don't commute.

But while you cannot diagonalize these things, you can find special states that are eigenstates of all of the elements of a permutation group. So you remember, when you say-- this is a very important point. Whenever you say you cannot simultaneously diagonalize two operators, it means that you cannot find a basis of states that are simultaneous eigenstates. But it may happen that you have one state that is an eigenstate of all these other things that don't commute. It is possible to have operators that don't commute and have one state that's an eigenstate of all of them. You cannot have a basis that is an eigenstate of all of them, because they don't commute.

But one state is possible. So we can find special states. Special states that are eigenstates of all permutation operators. So let's assume we have n particles, each living on v, in v, so that the n particles live on v tensor n. People write it like that, v tensor n, which is supposed to mean v tensor v with v appearing n times.

So here is a claim that we're going to postulate the existence of symmetric states. Those are the states that eventually will see are the ones physics ones. So postulate that there are the existence of symmetric states, psi s, n v tensor n. In the whole big space, there's symmetric states. And what is the characteristic of a symmetric state? The p alpha, any permutation. Remember, alpha means all these set of indices on psi s is equal to psi s for all alpha.

So the state is invariant. So we want to see that there is such a thing, states that are invariant, under all the permutation operators that we've constructed. This state would be eigenstates of all the permutation operators with eigenvalue equals to 1. So it's a simultaneous eigenvector of all these operators. But since the permutation operators don't commute, you cannot expect the basis.

So if there are symmetric states, they cannot form a basis in the full Hilbert space. There must be some smaller space. So we should be able to reach them by a projector into a subspace of symmetric states. How about defining now postulate anti-symmetric states. Psi A. P alpha on psi A should then be equal to-- what should I put? Negative psi A.

Well, yeah, that's the first thing we would put, but that's pretty problematic actually. So even when you postulate things, you know, postulate means, OK, we think they exist, then we'll try to build them. Here, we're trying to postulate that there are states that do this. There's a little bit of problems with this. First obvious problem, you say, oh well, this is a mathematical technicality. The identity element is supposed to be a permutation p 1, 2, 3. And the identity element is not going to change this one.

So that's not good. So suppose you have one transposition, and changes the state, a transposition should produce a minus sign, because it's anti-symmetric. But suppose you have now two transpositions. You act on them with two transpositions. One will change its sign. The other will change its sign. Now the total double transposition is a permutation operator. Shouldn't change the sign of the state.

So in fact, this is untenable. We're not even-- so even if we postulate something, we'd have to postulate something that makes some sense. And so far, it doesn't make sense. So what can we use? We can use the fact that there's some even permutations and some odd permutations. So we'll put the sign factor here, psi A. This is the only way to solve this problem is to put the sign factor epsilon sub alpha associated to the permutation.

Sometimes it's going to be a minus. Sometimes it's going to be a plus. For example, for the identity operator, it should be a plus. For a transposition, it should be a minus. So what is this epsilon alpha? Epsilon alpha is equal to 1 if p alpha is an even [? transpose ?] even or minus 1 if p alpha is odd.

So an odd permutation is one that has an odd number of transpositions. So that makes sense. This is a way to do this consistently. If you have a single transposition, and we'll put the minus, but if you have two transposition, it will put a plus, as it should. And the identity element is an even permutation. Therefore, it works as well.

So this is a nice thing. This is the only way you can define this anti-symmetric states, even before we construct them. So here are the names, and we'll stop and build them next time. So the symmetric state, symmetric states form a subspace of VN called sym N V. Symmetric in N states of V. The anti-symmetric states form a subspace of VN called anti N of V.

And our task for next time is to construct the projectors that bring you down to those spaces, analyze what are the properties of these spaces, and show that it solves the problem of exchange degeneracy, and that requires an extra postulate in quantum mechanics, a postulate for identical particles.