

8.044 SOLUTIONS EXAM #2

①

$$1. \quad P = P(T, \epsilon) \rightarrow dP = \left. \frac{\partial P}{\partial T} \right|_{\epsilon} dT + \left. \frac{\partial P}{\partial \epsilon} \right|_T d\epsilon$$

$$\left. \frac{\partial P}{\partial \epsilon} \right|_T = \left(a + \frac{b}{T} \right) N + 3cN\epsilon^2 \quad \{ \text{GIVEN} \}$$

$$\begin{aligned} \left. \frac{\partial P}{\partial T} \right|_{\epsilon} &= \frac{-1}{\left. \frac{\partial T}{\partial \epsilon} \right|_P \left. \frac{\partial \epsilon}{\partial T} \right|_P} = - \frac{\left. \frac{\partial P}{\partial \epsilon} \right|_T}{\left. \frac{\partial T}{\partial \epsilon} \right|_P} \leftarrow \{ \text{GIVEN} \} \\ &= \frac{\left[\left(a + \frac{b}{T} \right) N + 3cN\epsilon^2 \right] [dT^2 - b\epsilon]}{aT^2 + bT + 3cT^2\epsilon^2} = \frac{N}{T^2} [dT^2 - b\epsilon] \\ &= Nd - \frac{bN\epsilon}{T} \end{aligned}$$

$$P = \left(a + \frac{b}{T} \right) N\epsilon + cN\epsilon^3 + f(T) \quad \left\{ \begin{array}{l} \text{INTEGRATING FIRST} \\ \text{WITH RESPECT TO } \epsilon \end{array} \right\}$$

$$\left. \frac{\partial P}{\partial T} \right|_{\epsilon} = -\frac{bN\epsilon}{T^2} + f'(T) = Nd - \frac{bN\epsilon}{T^2} \Rightarrow f'(T) = Nd$$

$$f(T) = NdT + K$$

$$P(T_0, \epsilon=0) = P_0 = NdT_0 + K \Rightarrow K = P_0 - NdT_0$$

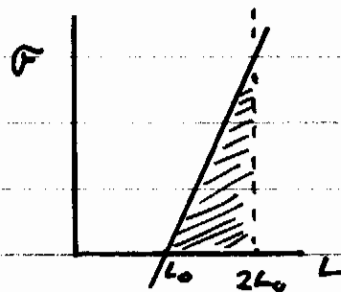
$$\underline{\underline{P = \left(a + \frac{b}{T} \right) N\epsilon + cN\epsilon^3 + Nd(T - T_0) + P_0}}$$

(2)

$$2. \quad a) \quad F(L_0, T) = 0 \quad F(2L_0, T) = (a + bT)L_0$$

$$\Delta W = \int_{L_0}^{2L_0} F dL$$

$$= \frac{1}{2} F(2L_0, T) L_0 = \underline{\underline{\frac{1}{2} (a + bT) L_0^2}}$$



$$b) \quad \Delta U = U(2L_0, T) - U(L_0, T) = \frac{a}{2} L_0^2$$

$$\Delta Q = \Delta U - \Delta W = \frac{a}{2} L_0^2 - \frac{1}{2} (a + bT) L_0^2 = \underline{\underline{-\frac{bT}{2} L_0^2}}$$

NOTE THAT HEAT FLOWS OUT OF THE ROD

$$c) \quad dU = \frac{\partial U}{\partial T}|_L dT + \frac{\partial U}{\partial L}|_T dL \quad \{ \text{EXPAND} \}$$

$$= dQ + F dL \quad \{ \text{1}^{\text{st}} \text{ LAW} \}$$

$$dQ = \underbrace{\frac{\partial U}{\partial T}|_L}_{cT^3} dT + \underbrace{\left(\frac{\partial U}{\partial L}|_T - F \right)}_{a(L-L_0) - a(L-L_0) - bT(L-L_0)} dL \quad \{ \text{REAR} \}$$

\{ FROM GIVEN \}

$$= 0 \Rightarrow cT^3 dT = bT(L-L_0) dL$$

$$\underline{\underline{\frac{dL}{dT} = \frac{cT^2}{b} \frac{1}{(L-L_0)}}}$$

∞ WHEN $L = L_0$,
CHANGES SIGN ABOVE
+ BELOW $L = L_0$

THIS AGREES WITH SUPPLIED SKETCH.

3. a) R GOES FROM 0 TO N.

MINIMUM E = -NJ (ALL PARALLEL)

MAXIMUM E = NJ (COMPLETELY STAGGERED)

b) IF THE FIRST SPIN IS UP, THERE ARE N! / ((N-R)! R!) DIFFERENT ARRANGEMENTS FOR A GIVEN R (ONE REVERSAL IS INDISTINGUISHABLE FROM ANOTHER). IF THE FIRST SPIN IS DOWN THERE ARE AN EQUAL NUMBER OF DIFFERENT ARRANGEMENTS.

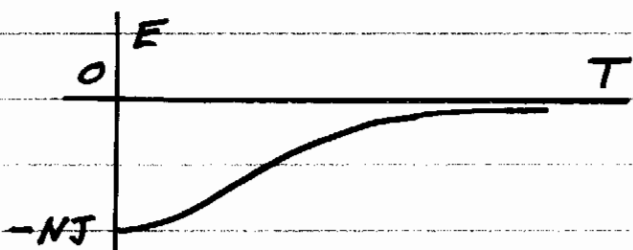
Omega(R) = 2N! / ((N-R)! R!)

S = k ln Omega approx k [ln 2 + N ln N - (N-R) ln (N-R) - R ln R]

1/T = dS/dE_N = dS/dR_N * dR/dE_N = k/2J [ln(N-R) - ln R]

2J/kT = ln(N-R/R) = ln(N/R - 1) => N/R = e^(2J/kT) + 1

R = N / (1 + e^(2J/kT)) E = NJ * (1 - e^(2J/kT)) / (1 + e^(2J/kT))



e) LIM R = N/2 as T approaches infinity

NOTE THAT THIS IS TRUE FOR BOTH POSITIVE & NEGATIVE J.

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8.044 Statistical Physics I
Spring 2013

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