8.044 Statistical Physics I   Spring Term 2013

Problem Set #7
Due in hand-in box by 12:40 PM, Wednesday, April 3

Problem 1: Free Expansion of a Gas

Before   After

A classical, monatomic, non-ideal gas has the equation of state

$$P(T,V) = \frac{NkT}{(V - bN)} - a\left(\frac{N}{V}\right)^2$$

where a and b are positive constants. The term $-bN$ adjusts the volume available to a given atom to compensate for the volume occupied by the other atoms. The term $-a\left(\frac{N}{V}\right)^2$ adjusts the pressure to take into account the long range (van der Waals) attractive interaction between the atoms. Neither of these corrections changes the constant volume heat capacity of the gas.

The gas is held in a container of negligible mass which is isolated from its surroundings. The gas is initially confined to 1/3 of the volume V_0 by a partition and is in thermal equilibrium at $T = T_i$. At $t = 0$ a hole is opened in the partition allowing the gas to expand irreversibly into the rest of the container, thus attaining a final volume V_0.

a) Which thermodynamic quantities are conserved in this process?

b) What is the new temperature of the gas when thermal equilibrium has finally been re-established ?

Problem 2 Use of a Carnot Cycle

Two identical bodies with temperature independent heat capacities C_0 are initially at different temperatures T_H and T_C. A Carnot cycle is run between them (with infinitesimal steps) until they have been reduced to a common temperature T_F.

a) Find T_F in terms of T_H and T_C.

b) Find the total work done on the outside world in this process. Is it positive or negative?

Problem 3: Cooling Liquid Helium

In a certain experiment liquid helium at an initial temperature of 1.0 K is to be cooled further by bringing it into contact with a paramagnetic salt which is initially at a lower temperature T_0. The heat capacity of the liquid may be expressed as $C_L = aT^3$ where $a = 128 \times 10^4$ ergs K^{-4}, and that of the salt may be expressed as $C_S = bT^{-2}$ where $b = 15 \times 10^4$ ergs K. Assume the mixture to be mechanically and thermally isolated from the rest of the universe and no work is done in the process.

a) What must be the initial temperature of the salt, T_0, if the final temperature of the system, helium plus salt, is to be 0.5 K?

b) Find the net change in the entropy of the universe during this process. To facilitate grading, and to minimize the chance of math errors, leave this answer in terms of the parameters a and b.

Problem 4: Torsional Pendulum
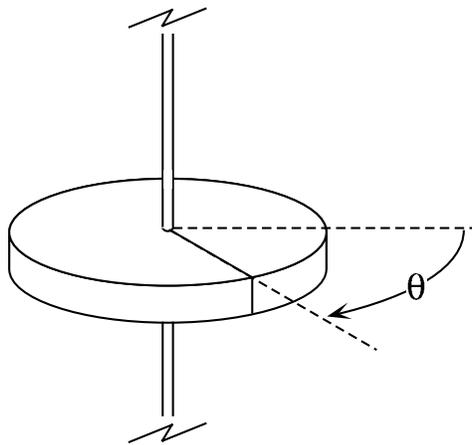
In order to measure a certain physical quantity it is necessary to know the equilibrium angle θ_0 of a torsional pendulum. θ_0 is determined by the minimum in the potential energy of the pendulum $V(\theta) = \frac{1}{2}K(\theta - \theta_0)^2$. The pendulum when rotating has a kinetic energy $T = \frac{1}{2}I\dot{\theta}^2$ where $\dot{\theta} \equiv d\theta/dt$. The precision with which θ_0 can be found is limited because the pendulum is in thermal equilibrium with its environment at a temperature T.

a) Find the root-mean-square uncertainty $< (\theta - \theta_0)^2 >^{1/2}$ associated with a measurement of θ due to classical thermal noise.

b) Find $< \theta\dot{\theta} >$.

Problem 5: The Hydrogen Atom

The simplest model of hydrogen has bound electronic energy eigenstates $|n, l, m_l >$ with energies $\epsilon_{n,l,m_l} = -A/n^2$ where $A = 13.6$ eV.

a) Find the ratio of the number of atoms in the first excited energy level to the number in the lowest energy level at a temperature T. [Hint: Be careful of the difference between energy levels and states of the system.] Evaluate this for $T = 300K$ and $T = 1000K$.

b) To find the actual fraction of atoms in any given state one needs the partition function. Show that the partition function diverges, even when the unbound states are neglected.

c) Any ideas why statistical mechanics does not seem to work for this, one of nature's simplest systems?

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