PROFESSOR: SHO algebraically. And we go back to the Hamiltonian, p squared over 2m plus 1/2 m omega squared x hat squared. And what we do is observe that this some sort of sum of squares plus p squared over m-- p squared over m squared omega squared. So the sum of two things squared. Now, the idea that we have now is to try to vectorize the Hamiltonian.

And what we call vectorizing is when you write your Hamiltonian as the product of two vectors, V times W. Well actually, that's not quite the vectorization. You want kind of the same vector, and not even that. You sort of want this to be the Hermitian conjugate of that. And if there is a number here, that's OK. Adding numbers to a Hamiltonian doesn't change the problem at all. The energies are all shifted, and it's just how you're defining the zero of your potential, is doing nothing but that.

So vectorizing the Hamiltonian is writing it in this way, as V dagger V. And you would say, why V dagger V? Why not VV dagger or VV or V dagger V dagger? Well, you want the Hamiltonian to be Hermitian. And this thing is Hermitian. You may recall that AB dagger. The Hermitian conjugate of AB dagger is B dagger A dagger. So the Hermitian conjugate of this product is V dagger times the dagger of V dagger. A dagger of a dagger is the same operator, when you dagger it twice, you get the same.

So this is Hermitian. V dagger times V is a Hermitian operator, and that's a very good thing. And there will be great simplifications. If you ever succeed in writing a Hamiltonian this way, you've gone 90% of the way to solving the whole problem. It has become infinitely easier, as you will see in a second, if you could just write this vectorization.

So if you had x minus-- x squared minus this, you would say, oh, clearly that's-- A squared minus B squared is A minus B time A plus B, but there's no such thing here. It's almost like A squared plus B squared. And how do you sort of factorize it? Well, actually, since we have complex numbers, this could be A minus IB times A plus IB. That is correctly A squared plus B squared, and complex numbers are supposed to be friends in quantum mechanics, so having Is, there's probably no complication there.

So let's try that. I'll write it. So here we have x squared plus p squared over m squared omega squared. And I will try to write it as x minus i p hat over m omega times x plus I p hat over m omega. Let's put the question mark before we are so sure that this works. Well, some things work. The only danger here is that these are operators and they don't commute. And when we do this, in one case, in the cross-terms, the A is to the left of B, but the other problem the B is to the left of A. So we may run into some trouble. This may not be exactly true.

So what is this? This x with x, fine. x squared. This term, p with ps, correct. Plus p squared over m squared omega squared. But then we get plus i over m omega, x with p minus p with x, so that x, p commutator. So vectorization of operators in quantum mechanics can miss a few concepts because things don't commute. So the cross-terms give you that, and this x, p is I h bar, so this whole term will give us the following statement.

What we've learned is that what we wanted, x squared plus p squared over m squared omega squared is equal to-- so I'm equating this line to the top line-- is equal to x hat minus i p hat over m omega times x hat plus i p hat over m omega. And then, from this whole term, i with i is minus, so it's h bar over m omega. So I'll put it in-- it's a minus h bar over m, [INAUDIBLE]. So here is plus h bar over m omega times a unit vector, if you wish.

OK. So this is very good. In fact, we can call this V dagger and this V. Better call this V first and then ask, what is the dagger of this operator? Now, you may remember that, how did we define daggers? If you have phi with psi and the inner product-- with an integral of five star psi-- if you have an A psi here, that's equal to A dagger phi psi.

So an operator is acting on the second wave function, moves as A dagger into the first wave function. And you know that x moves without any problem. x is Hermitian. We've discussed that p is Hermitian as well, moves to the other side. So the Hermitian conjugate of this operator is x, the p remains means p, but the i becomes minus i. So this is correct. If this second operator is called V, the first operator should be called V dagger. That is a correct statement. One is the dagger of the other one.

So the Hamiltonian is 1/2 m omega squared times this sum of squares, which is now equal to V dagger V plus h bar over m omega. So h hat is now 1/2 m omega squared V dagger V plus a sum, which is plus 1/2 h bar omega. So we did it. We vectorized the Hamiltonian V dagger V, and this is quite useful.

So the Vs, however, have units. And you probably are aware that we like things without units, so that we can see the units better. This curve is perfectly nice. It's a number added to the

Hamiltonian. It's h omega, it has units of energy, but this is still a little messy. So let's try to clean up those Vs, and the way I'll do it is by computing their commutator, to begin with.

So let's compute the commutator of V and V dagger and see how much is that commutator. It's a simple commutator, because it involves vectors of x and V. So it's the commutator of x plus ip over m omega, that's V, with x minus ip over m omega. So the first x talks only to the second piece, so it's minus i over m omega x, p. And for the second case, you have plus i over am omega p with x. This is i h bar, and this is minus i h bar. Each term will contribute the same, i times minus i is plus, so h bar over and, omega times the 2. That is V dagger V. VV dagger, I'm sorry. 2 h bar over m omega.

So time to change names a little bit. Let's do the following. Let's put square root of m omega over 2 h bar V. Have a square root of m omega over 2 h bar V dagger, commute to give you 1. That's a nice commutator. It's one number-- or an operator is the same thing. So I brought the square root into each one.

And we'll call the first term-- because of reasons we'll see very soon-- the destruction operator, A square root of m omega over 2 h bar V. It's called the destruction operator. And the dagger is going to be A dagger. Some people put hats on them. I sometimes do too, unless I'm too tired. 2h bar V dagger. And those A and A daggers are now unit-free-- and you can check That-- Because they have the same units. And A with A dagger is the nicest commutator, 1.

Is A a Hermitian operator? Is it? No. A is not Hermitian. A dagger is different from A. A is basically this thing, A dagger is this thing. So not Hermitian. So we're going to work with these operators. They're non-Hermitian. I need to write the following equations. It's very-- takes a little bit of writing, but they should be recorded, they will always make it to the formula sheet. And it's the basic relation between A, A dagger, and x and p. A is this, A dagger, as you know, is x minus ip hat over m omega. Since I'm copying, I'd better copy them right. x, on other hand, is the square root of h bar over 2m omega A plus A dagger, and p is equal to i square root of m omega h bar over 2 A dagger minus A.

So these four equations, A and A dagger in terms of x and p and vice versa, are important. They will show up all the time. Here are the things to notice. A and A dagger is visibly clear that on is the Hermitian conjugate of the other. Here, x is Hermitian. And indeed, A plus A dagger is Hermitian. When you do the Hermitian conjugate of A plus A dagger, the first A becomes an A dagger. The second A, with another Hermitian conjugation, becomes A. So this is Hermitian. But p is Hermitian, and here we have A dagger minus A. This is not Hermitian, it changes sign. Well, the i is there for that reason, and makes it Hermition. So there they are, they're Hermitian, they're good.