PROFESSOR: This is a wonderful differential equation, because it carries a lot of information. If you put this psi, it's certainly going to be a solution. But more than that, it's going to tell you the relation between k and omega. So if you try your-- we seem to have gone around in circles.

But you've obtained something very nice. First, we claim that that's a solution of that equation and has the deep information about it. So if you try again, psi equal e to the ikx minus i omega t, what do we get?

On the left hand side, we get ih minus i omega psi. And on the right hand side, we get minus h squared over 2m and two derivatives with respect to x. And that gives you an ik times and other ik. So ik squared times psi.

And the psis cancel from the two sides of the equation. And what do we get here? h bar omega. It's equal to h squared k squared over 2m, which is e equal p squared over 2m.

So it does the whole job for you. That differential equation is quite smart. It admits these as solutions.

Then, this will have definite momentum. It will have definite energy. But even more, when you try to see if you solve it, you find the proper relation between the energy and the momentum that tells you you have a particle.

So this is an infinitely superior version of that claim that that is a plane wave that exists. Because for example, another thing that you have here is that this equation is linear. Psi appears linearly, so you can form solutions by superposition.

So the general solution, now, is not just this. This is a free particle Schrodinger equation. And you might say, well, the most general solution must be that, those plane waves. But linearity means that you can compose those plane waves and add them. And if you can add plane waves by Fourier theorem, you can create pretty much all the things you want.

And if you have this equation, you know how to evolve free particles. Now, you can construct a wave packet of a particle and evolve it with the Schrodinger equation and see how the wave packet moves and does its thing. All that is now possible, which was not possible by just saying, oh, here is another wave.

You've worked back to get an equation. And this is something that happens in physics all the time. And we'll emphasize it again in a few minutes. You use little pieces of evidence that lead you-- perhaps not in a perfectly logical way, but in a reasonable way-- to an equation.

And that equation is a lot smarter than you and all the information that you put in. That equation has all kinds of physics. Maxwell's equations were found after doing a few experiments. Maxwell's equation has everything in it, all kinds of phenomena that took years and years to find.

So it's the same with this thing. And the general solution of this equation would be a psi of x and t, which would be a superposition of those waves. So you would put an e to the ikx minus i omega t. I will put omega of k because that's what it is.

Omega is a function of k. And that's what represents our free particles-- omega of kt. And this is a solution. But so will be any superposition of those solutions. And the solutions are parametrized by k. You can choose different momenta and add them.

So I can put a wave with one momentum plus another wave with another momentum, and that's perfectly OK. But more generally, we can integrate. And therefore, we'll write dk maybe from minus infinity to infinity. And we'll put a phi of k, which can be anything that's not part of the differential equation.

Now, this is the general solution. You might probably say, wow, how do we know that? Well, I suggest you try it. If you come here, the ddt will come in. We'll ignore the k. Ignore this. And just gives you the omega factor here.

That ddx squared-- we'll ignore, again, all these things, and give you that. From the relation omega minus k equals 0, you'll get the 0. And therefore, this whole thing solves the Schrodinger equation-- solves the Schrodinger equation.

So this is very general. And for this, applies what we said yesterday, talking about the velocity of the waves. And this wave, we proved yesterday, that moves with a group velocity, v group, which was equal to d omega dk at some k0, if this is localized at k0.

Otherwise, you can't speak of the group velocity this thing will not have a definite group velocity. And the omega dk-- And you have this relation between omega and k, such a to way that this is the evp, as we said yesterday. And this is ddp of b squared over 2m, which is p over m, which is what we call the velocity of the particle.

So it moves with the proper velocity, the group velocity. That's actually a very general solution. We'll exploit it to calculate all kinds of things. A few remarks that come from this equation.

Remarks. 1, psi cannot be a real. And you can see that because if psi was real, the right hand side would be real. This derivative would be real because the relative of a real function is a real function. Here you have an imaginary number.

So structurally, it is forbidden to have full wave functions that are real. I call these full wave functions because we'll talk sometime later about time independent wave functions. But the full wave function cannot be real.

Another remark is that this is not the wave equation of the usual type-- not a usual wave equation. And what a usual wave equation is something like d second phi dx squared minus 1 over v squared d second phi dt squared equals zero. That's a usual wave equation.

And the problem with that wave equation is that it has real solutions. Solutions, phi that go like functions of x minus is vt, plus minus x over vt. And we cannot have those real solutions.

So we managed to get a wave, but not from a usual wave equation. This, waves also all move with some same velocity, velocity, v, of the wave. These waves don't do that. They have a group velocity.

It's a little bit different situation. And what has happened is that we still kept the second derivative, with respect to x. But in time, we replaced it by first derivative. And we put an i. And somehow, it did the right job for us.