PROFESSOR: The simplest quantum system. In order to decide what could be the simplest quantum system you could say a particle in a box. It's very simple, but in a sense it's not all that simple. It has infinitely many states. All these functions on an interval, and then the energy is where infinitely many of them, so not that simple. OK, infinite bound says something with one bound. OK, a delta function potential just one bound state, but it has infinitely many scattering states. It's still complicated. What could be simpler? Suppose you have the Schrodinger equation. H psi. And we work in general we know that this thing has energy eigenstates, and probably we should focus on them. So Psi equal E to the minus I, Et over H bar. Little psi, and then have H Psi equal E psi.

That is quantum mechanics, and you could say, well it's up to me to decide what the Hamiltonian is. If I want to invent the simplest quantum mechanical system. On the other hand, there are some things that should be true. These are complex numbers, energies, H must be an operator that has units of energy. And we also saw that if we want probabilities that are going to be associated with PSI squared to be conserved we need H to be Hermitian. There should be some notion of inner product. Some sort of operation that gives us numbers we used to defy PSI that gives a number. To complex numbers in general, and has the property of somewhat conjugates this thing. It has this, and integrates, but maybe if you're doing the simplest quantum mechanical system in the world it will be simpler than an integral. Integrals are complicated.

But anyway we have something like that, and we want H to be Hermitian. Let me write this in for any operator A, this is equal to a dagger Psi. And that's a Hermitian conjugate. That's a general definition, and we want H to be Hermitian. H dagger equal H. OK, in some sense you could say that's quantum mechanics for you. It's a Schrodinger equation, a Hamiltonian, an inner product, a notion of Hermitian operators, and then you're supposed to solve it. And what we've done is solve this for a whole semester, and try to understand some physics out of it.

But we started with the notion that something simple would be a particle living in one dimension, and that's a very reasonable thought. Motivated from classical mechanics that surely we have particles that move, and moving in three dimensions is more complicated. We waited towards the end of the semester to do three dimensions, but moving in one dimension is already kind of interesting, and complicated. We had Psi of X that represented the fact that the particle could be anywhere here. How can I simplify this? The key to simplifying this is maybe not to be too attached to the physics for a while, and try to visualize what could you describe that was simpler.

Suppose the particle could only live at two points X1, and X2. The particle can be here, or here. Now we've re-aligned down to just two points. It can only be this point, or that point. And you say, that's very in physical. But let's wait a second, and think of this. What does that mean? We used to have Psi effects that could be anywhere, and we wrote it as a function. If I think of this the simplest thing OK, the simplest thing is a particle is just at one point. There is only one point. The whole world for the particle is one point, and it's there. But that probably is not too interesting because the particle is there. The probability defined there is always one, and what can you do with It?

But if you have two points there's room for funny things to happen. We'll assume that the particle can be in two points. From F of this Psi effects will go to a new Psi effects that has two pieces of information. The value of PSI at x1, and the value of Psi at X2. And those are two numbers alpha, and beta. Alpha squared would be the probability to be at the X1. Beta squared would be the probability to be at X2. And this may remind you already of something we're doing with interferometers. In which the photon could be in the upper branch, or the lower branch, and you have two numbers. This is somewhat analogous except that the interferometer you could eventually put more beam splitters, and maybe later three branches, or four branches, or things like that.

Here I want to consider two things, particle there. One thing that this could be strictly that, but now let's relax our assumptions. It could also mean for example, if you have a box, and a partition. And there's the left side of the box, and the right side of the box. And the molecule can either be on the left side, or on the right side. That's a fairly physical question. Here you could be probability the amplitude to be on the left, or amplitude to be on the right. Two component vector just like that. One would be the amplitude to be in either one, and maybe that amplitude changes in time. Or it could be that you have a particle, and suddenly you discovered that yeah, the particle is at rest. It's not moving. It's not doing anything. It's one single point, not two points. But actually this particle has maybe something called spin, and the spin can be up, or the spin can be down.

We it could invent something. We could call it spin, or a particle could be in this state, or in that state. And if that's possible for a particle you could have here the amplitude for up spin, and the amplitude for down spin. And those would be the two numbers. It's lots of possibilities in the sense this is a classic problem waiting for a physical application in quantum mechanics. Let's push it a little more. Now how would we do inner products? We decided OK, you need to do inner products. And what was the inner product of two functions phi and psi was the integral, the X of phi star of X1 times-- phi star of X times psi of X.

And what you're really doing is taking the values of the first wave function at one point. Complex conjugating it, take the value of the second wave function at the same point complex conjugating it. If you would have two vectors like this alpha, and beta the first wave function. Alpha one, beta one, and the second wave function. Alpha two, beta two. The inner products psi 1, psi 2 should be the analog of this thing which is multiply things at the same point. You should do the alpha one star. That's alpha two plus beta one star times beta two. That would be the nice way to do this.

You could think of this as having transposed this alpha one, and complex conjugated it. Beta one, and then the matrix product with alpha one, beta one. You transpose complex conjugate the first, and you do that with the second. When you study a little more quantum mechanics in 805 you will explore this analogy even more in that you will think of a wave function as a column vector, infinite one. psi at zero, psi at epsilon, psi at two epsilon, psi at minus epsilon. So you've sliced the x-axis and conserved an infinite vector. And that's all wave function. It's not so unnatural to do this, and this will be our inner product.

How about H be in Hermitian. That just means for matrices that H transpose complex conjugate that dagger, Hermitian, is equal to H. And you may have seen that that's what dagger means. You transpose a complex conjugate. If you haven't seen it you could prove it now using this rule for the inner product because the inner product will tell you how to construct the dagger of any operator. And you will find that indeed the dagger what it does is transposes, and complex conjugates it. And it sort of comes because the inner product transposes, and complex conjugates the first object.