PROFESSOR: Solutions can be organized. One way to do it, it's not the standard way, is with plotting n here and l here, and you have 0, 1, 2, 3, 1, 2, 3. You have all this points. Remember, every point this allowed. Every integer combination is allowed.

All those are energy levels of the hydrogen atom, but now you can see that this point, when both are 0, corresponds to n equal to 1. Because if n and l are 0, n is equal to 1, and there's just one solution with n equal to 1. These two points here, when l is equal to 1 and n equals 0, with n is equal 1 and l equals 0, represent the two possibilities that realize n equals to 2. n equals to 2 is realized by having 1 and 0 or 1 and 0, two values.

Similarly, there are three things with n equal to 3, four things-- my graph is not that great-- with n equal to 4, and more and more states. So for each n you have that n plus l is equal to n minus 1. And l can never exceed n minus 1, and that's physically quite something that people remember. But also n cannot exceed n minus 1 or 0, both are there, limited by these quantities.

So if you have some n, you will have l and n, for example, for some quantum number n. You will have l equals 0 and n would be n minus 1. That would work out. l plus n would be n minus 1, or 1 and n minus 2, or all the things up to n minus 1 and 0. So they take turns. They have to add up to n minus 1.

So let's plot. Let's actually, we can go here. We don't have too much more to say at this moment. So let's have this. We can do a little counting that is interesting, and I'll count the number of states for a given n.

So for example, for n equals to 1, what can we have? We said, l equals 0, and capital N is equal to 0 as well, and that's one state for n equals 2. What can you have? n equals to 2, you could have l equals 1, or l equals 0. So l equals 1, or l equals 0.

And how many states do we have here? Well, l equals 1 can have m equals 1, 0, and minus 1. Remember, the m values is another label for states. Those are different states. So here, there are 3 states, plus 1 state, 3 plus 1 states, that's 4. n equals 3 will have l equals 2, l equals 1, and l equals 0, which is 5 states plus 3 states plus 1 state, which is 9 states, and that's 3 squared. And 4 was actually 2 squared, and if you go on to n you will have n square states. Something that perhaps you could try to count, and show that that's true.

So what are our quantum numbers for the states of hydrogen? Well, our quantum numbers are, it's our choice, but physically we want to understand them each intuitively. So here we go. One most important quantum number, and its name says so, is principle quantum number. So the quantum numbers of hydrogen, and the first important thing is n. We definitely cannot do away with n. It fixes our energies. And now we have a possibility.

We look at this and we say, yeah well, I actually I either need to determine what is l or what is n. So it doesn't even come close. Physicists will not say, oh, I want to describe the quantum number by the degree of the polynomial inside the solution. No, physicists will say, I want to use the angular momentum. And certainly, if you know l, and you know n, you know capital N.

So capital N is a funny number. It has to do with the degree of the polynomial that shows up in between this leading behavior and that exponential behavior, very interesting, but not directly physical. The l, however, is directly associated to an observable angular momentum. So to describe the state that I have here, if I give you n and l, you can see that you determine which state you are.

So the second quantum number is going to be l, and the third quantum number is unavoidable. It's the z component of angular momentum, should be m. That's also physical. We should not skip it. So these are our quantum numbers, and they fix capital N, in case you're interested, as n minus l plus 1, and that's interesting information.

So let's recall our variables. OK, a rho is here. That's very nice. So rho is 2 kappa z over a naught r, but now we know what kappa is. Kappa is 1 over 2n. So actually, the rho variable is tailored to the quantum numbers. It's just Zr over n a naught, where n is the principle quantum number.

So back to the solution. You see, we have to recap quickly. Psi nlm is equal to U of the energy of the radial equation-- so n and l is sufficient for that-- over r Ylm. Or the U is the thing that we had here, Ul, and now it has an energy into it. So it's a rho to the l plus 1, still r and rho up to numbers. So this is like. A rho, a Wnl if we wish, of rho e to the minus rho, and Ylm theta phi.

Well, let's write one more equation, and then finish. So just to give the feeling of this solution, what does that give you? Rho to the l, a polynomial of rho, which is a polynomial of degree n. n, which is little n minus l plus 1 times e to the minus rho Ylm. It's important for you to see the whole solution. This is the whole solution of the hydrogen atom.

I'll write it in one more way. A, a constant, because this is similar. Rho, well, rho is, in terms of units, at least has r over a naught to the l. Here it is a polynomial in r over a naught of degree. Little n minus l plus 1, and this polynomial, we could make a whole study of it.

These are Laguerre polynomials. We will not look into them in this course. You may do it in a more advanced course. It's interesting, but it's better to just get an intuition as to what's happening here. There is an e to the minus rho, which is interesting to have fully.

So this is e to the minus Zr over n a naught, and there's a Ylm of theta and phi. So this is your whole solution for the hydrogen atom. We should write the simplest one psi 1, 0, 0, n equals 1, l equals 0, m equals 0, spherically symmetric. Here it is, 1 over pi a cube, e to the minus r over a naught For the KZ is equal to 1. Ground state of hydrogen.