PROFESSOR: Mach-Zehnder--

interferometers.

And we have a beam splitter.

And the beam coming in, it splits into 2. A mirror--

another mirror. The beams are recombined into another beam splitter. And then, 2 beams come out. One to a detector d0-- and a detector d1.

We could put here any kind of devices in between. We could put a little piece of glass, which is a phase shifter. We'll discuss it later. But our story is a story of a photon coming in and somehow leaving through the interferometer.

And we want to describe this photon in quantum mechanics. And we know that the way to describe it is through a wave function. But this photon can live in either of 2 beams. If a photon was in 1 beam, I could have a number that tells me the probability to be in that beam.

But now, it can be in either of 2 beams. Therefore, I will use two numbers. And it seems reasonable to put them in a column vector. Two complex numbers that give me the probability amplitudes--

for this photon to be somewhere. So you could say, oh, look here. What is the probability that we'll find this photon over here? Well, it may depend on the time. I mean, when the photon is gone, it's gone.

But when it's crossing here, what is the probability? And I have 2 numbers. What is the probability here, here, here, here? And in fact, you could even have 1 photon that is coming in through 2 different channels, as well.

So I have 2 numbers. And I want, now, to do things in a normalized way. So this will be the probability amplitude to be here. This is the probability amplitude to be down. And therefore, the probability to be in the upper one-- you do norm squared.

The probability to be in the bottom one, you do norm squared. And you get 1. Must get 1. So if you write 2 numbers, they better satisfy that thing. Otherwise, you are not describing probabilities.

On the other hand, I may have a state that is like this. Alpha-- oh, I'll mention other states. State 1-0 is a photon in the upper beam.

No probability to be in the lower beam. And state 0-1 is a photon in the lower beam.

So these are states. And indeed--

think of superposition. And we have that the state, alpha beta-- you know how to manipulate vectors-- can be written as alpha 1-0. Because the number goes in and becomes alpha 0. Plus beta 0-1.

So the state, alpha beta, is a superposition with coefficient alpha of the state in the upper beam plus the superposition with coefficient beta of the state in the lower beam. We also had this little device, which is called the beam shifter of face delta.

If the probability amplitude completing is alpha to the left of it, it's alpha e to the i delta to the right of it, with delta a real number. So this is a pure phase. And notice that alpha is equal to e to the i delta.

The norm of a complex number doesn't change when you multiply it by a phase. The norm of a complex number times a phase is the norm of the complex number times the norm of the phase. And the norm of any phase is 1.

So actually, this doesn't absorb the photon, doesn't generate more photons. It preserves the probability of having a photon there, but it changes the phase.

How does the beam splitter work, however? This is the first thing we have to model here. So here is the beam splitter.

And you could have a beam coming--

A 1-0 beam hitting it. So nothing coming from below. And something coming from above. And then, it would reflect some and transmit some. And here is a 1-- is the 1 of the 1-0. And here's an s and a t.

Which is to mean that this beam splitter takes the 1-0 photon and makes it into an st photon. Because it produces a beam with s up and t down.

On the other hand, that same beam splitter-- now, we don't know what those numbers s and t are. That's part of designing a beam splitter. You can ask the engineer what are s and t for the beam splitter.

But we are going to figure out what are the constraints. Because no engineer would be able to make a beam splitter with arbitrary s and t.

In particular, you already see that if 1-0-- if a photon comes in, probability conservation, there must still be a photon. You need that s squared plus t squared is equal to 1 because that's a photon state.

Now, you may also have a photon coming from below and giving you uv. So this would be a 0-1 photon, giving you uv.

And therefore, we would say that 0-1--

gives you uv. And you would have u plus v norm squared is equal to 1. So we need, apparently, 4 numbers to characterize the beam splitter. And let's see how we can do that.

Well, why do we need, really, 4 numbers? Because of linearity. So let's explore that a little more clearly. And suppose that I ask you, what happens to an alpha beta state--

alpha beta state if it enters a beam splitter? What comes out? Well, the alpha beta state, as you know, is alpha 1-0 plus beta 0-1. And now, we can use our rules.

Well, this state, the beam splitter is a linear device. So it will give you alpha times what it makes out of the 1-0. But out of the 1-0 gives you st.

And the beta times 0-1 will give you beta uv. So this is alpha s plus beta u times alpha t plus beta v. And I can write this, actually, as alpha beta times the matrix, s u t v.

And you get a very nice thing, that the effect of the beam splitter on any photon state, alpha beta, is to multiply it by this matrix, s u t v. So this is the beam splitter. The beam splitter acts on any photon state. And out comes the matrix times the photon state.

This is matrix action, something that is going to be pretty important for us.

How do we get those numbers? After all, the beam splitter is now determined by these 4 numbers and we don't have enough information. So the manufacturer can tell you that maybe you've got-- you bought a balanced beam splitter.

Which means that if you have a beam, half of the intensity goes through, half of the intensity gets reflected. That's a balanced beam splitter. That simplifies things because the intensity here, the probability, [INAUDIBLE] must be the same as that.

So each norm squared must be equal to 1/2, if you have a balanced--

beam splitter.

And you have s squared equal t squared equal u squared equal v squared equal 1/2. But that's still far from enough to determine s, t, u, and v. So rather than determining, them at this moment, might as well do a guess.

So can it be that the beam splitter matrix-- Could it be that the beam splitter matrix is 1 over square root of 2, 1 over square root of 2, 1 over the square root of 2, and 1 over square root of 2. That certainly satisfies all of the properties we've written before.

Now, why could it be wrong? Because it could be pluses or minuses or it could be i's or anything there. But maybe this is right. Well, if it is right, the condition that it be right is that, if you take a photon state, 1 photon-- after the beam splitter, you still have 1 photon.

So conservation of probability. So if you act on a normalized photon state that satisfies this alpha squared plus beta squared equal 1, it should still give you a normalized photon state. And it should do it for any state.

And presumably, if you get any numbers that satisfy that, some engineer will be able to build that beam splitter for you because it doesn't contradict any physical principle. So let's try acting on this with on this state-- 1 over square root of 2, 1 over square root of 2.

Let's see. This is normalized-- 1/2 plus 1/2 is 1. So I multiply. I get 1/2 plus 1/2 is 1, and 1. Sorry, this is not normalized. 1 squared plus 1 squared is 2, not 1. So this can't be a beam splitter. No way.

We try minus 1 over square root of 2. Actually, if you try this for a few examples, it will work. So how about if we tried in general. So if I try it in general, acting on alpha beta, I would get 1 over square root of 2 alpha plus beta and alpha minus beta.

Then, I would check the normalization. So I must do norm of this 1 squared. So it's 1/2 alpha plus beta squared plus 1/2 alpha minus beta norm squared.

Well, what is this? Let me go a little slow for a second. [INAUDIBLE] plus beta star.

Plus alpha minus beta. Alpha star minus beta star.

Well, the cross terms vanish. And alpha alpha star, alpha alpha star, beta beta star, beta beta star add. So you do get alpha squared plus beta squared. And that's 1 by assumption because you started with a photon.

So this works. This is a good beam splitter matrix.

It does the job. So actually--

Consider this beam splitters. Actually, it's not the unique solution by all means.

But we can have 2 beam splitter that differ a little bit. So I'll call beam splitter 1 and beam splitter 2. Beam splitter or 1 will have this matrix. And beam splitter 2 will have the matrix were found here, which is a 1 1 1 minus 1.

So both of them work, actually. And both of them are good beam splitters. I call this--

beam splitter 1.

And this, beam splitter 2. And we'll keep that. And so we're ready, now, to think about our experiments with the beam splitter.