BARTON ZWIEBACH: Let's talk now about entanglement. So we talk about entanglement when we have two non-interacting particles. You don't need a strong interaction between particles to produce entanglement, the particles can be totally non-interacting. Suppose particle 1 can be in any of these states-- u 1, u 2. Let's assume just u 1 and u 2. And particle 2 can be in states v 1 and v 2.

And you have these two particles flying around, these are possible states of particle 1 and possible states of particle 2. Now you want to describe the full system, the quantum state of the two particles. States of the two particles. Two particles. Well, it seems reasonable that to describe the state of the two particles that are not interacting, I should tell you what particle 1 is doing and what particle 2 is doing.

OK, so particle 1 could be doing this. Could be u 1. And particle 2 could be doing v 1. And in a sense, by telling you that, we've said what everything is doing. Particle 1 is doing u 1, particle 2 is doing u 2. And mathematically, we like to make this look like a state and we want to write it in a coherent way. And we sort of multiply these two things, but we must say sort of multiply, because this strange multiplication, this, you know, we think of them as vectors or states, so how do you multiply states? So you put something called the tensor product, a little multiplication like this.

So you could say, don't worry, it's kind of like a product, and it's the way we do it. We don't move things across, the first state here, the second state here, and that's a possible state.

Now, I could have a different state. Because particle 1, in fact, could be doing something a little different. Could be doing alpha 1 u 1 plus alpha 2 u 2, and maybe particle 2 is doing beta 1 v 1 plus beta 2 v 2. And this would be all right. I'm telling you what particle 1 is doing and I'm telling you what particle 2 is doing and the rules of tensor multiplication or this kind of multiplication to combine those states are just like a product, except that as I said, you never move the states across. So you just distribute, so you have alpha 2, beta 1, the number goes out, u 1 v 1-- that's the first factor-- plus alpha 1 beta 2 u 1 v 2 plus alpha 2 beta 1 u 2 v 1 plus alpha 2 beta 2 u 2 v 2. I think I got it right. Let me know. I just multiplied and got the numbers out. The numbers can be move out across this product.

OK, so that's a state and that's a superposition of states, so actually, I could try to write a different state now. You see, we're just experimenting, but here is another state. u 1 v 1 plus u 2 v 2. Now this is a state that actually seems different. Quite different. Because I don't seem to be able to say that what particle 1 is doing and what particle 2 is doing separately. You see, I can say when particle 1 is doing u 1, particle 2 is doing v 1. And if when particle 1 is doing u 2, this is v 2. But can I write this as some state of the first particle times some state of the second particle?

Well, let's see. Maybe I can and can write it in this form. This is the most general state that you can say, particle 1 is doing this, and particle 2 is doing that. So can they do that? Well, I can compare these two terms with those and they conclude that alpha 1 beta 1 must be 1. Alpha 2 beta 2 must be also 1. But no cross products exist, so alpha 1 beta 2 must be 0 and alpha 2 beta 1 must be 0. And that's a problem because either alpha 1 is 0, which is inconsistent, or beta 2 is 0, which is inconsistent with that, so now this state is un-factorizable. It's a funny state in which you cannot say that this quantum state can be described by telling what the first particle is doing and what the second particle is doing. What the first particle is doing depends on the second and what the second is doing depends on the first. This is an entangled state.

And then we can build entangled states and our very strange states. So with two particles with spins, for example, we can build an entangled states of 2 spin 1/2 particles. And this state could look like this-- the first particle is up along z and the second particle is down along z, plus a particle that is down along z for the first particle, but the second is up along z.

And these are 2 spin 1/2 particles and in the usual notation, these experiments in quantum mechanics and black hole physics, people speak of Alice and Bob. Alice has one particle, Bob has the other particle. Maybe Alice is in the moon and has her electron and Bob is on earth and has his electron, and the two electrons, one on the earth and in the moon, are in this state. So then we say that Alice and Bob share an entangled pair. And all kinds of strange things happen. People can do those things in the lab-- not quite one in the earth and one in the moon, but one photon at one place and another photon entangled with it at 100 kilometers away, that's pretty doable.

And they are in this funny state in which their properties are currently that in surprising ways. So what happens here? Suppose Alice goes-- or let's say Bob goes along and measures his spin and he finds his spin down. So-- oh, you look here, oh, here is down for Bob. So at this moment, the whole state collapses into this. Because up with Bob didn't get realized. So once Bob measures and he finds down, the whole state goes into this. So if Alice-- on the moon or in another galaxy-- at that instant looks at her spin, she will find it's up before light has had time to get there. Instantaneously. It will go into this state.

People were sure somehow this violates special relativity. It doesn't. You somehow when you think about this carefully, you can't quite send information, but the collapse is instantaneous in quantum mechanics. Somehow, Bob and Alice cannot communicate information by sharing this entangled pair, but it's an interesting thing why it cannot happen.

Einstein again objected to this. And he said, this is a fake thing. You guys are going to share-- and now, of course, they have to share many entangled pairs to do experiments, so maybe 1,000 entangled pairs. And Einstein would say, no, that's not what's happening. What's happening is that some of your entangled pairs are this. That is, Bob is down, Alice is up, some of them are this-- and there's no such thing as this entanglement and indeed, if you measure and you find down, she will find up, and if you measure and you find up, she will find down, and there's nothing too mysterious here.

But then came John Bell in 1964 and discovered his Bell inequalities that demonstrated that if Alice and Bob can measure in three different directions, they will find correlations that are impossible to explain with classical physics. It took a lot of originally for Bell to discover this, that you have to measure in three directions, and therefore, the kind of correlations that appear in entangled states are very subtle and pretty difficult to disentangle. So that's why entanglement is a very peculiar subject. People think about it a lot because it's very mysterious. It somehow violates classical notions, but in a very subtle way.