PROFESSOR: So here is of something funny. You might say, OK, what is simpler? A theory that is linear or a theory that is not linear? And the answer, of course, a linear theory is much simpler. General-- Maxwell's equations are linear. Einstein's theory of relativity is very nonlinear, very complicated.

How about classical mechanics? Is classical mechanics linear or nonlinear? What do we think? Can't hear anyone. Linear, OK. You may think it's linear because it's supposed to be simple, but it's not. It's actually is very nonlinear. Newton could solve the two body problem but he couldn't solve the three body problem. Already with three bodies, you cannot superpose solutions that you get with two bodies. It's extraordinarily complicated, classical mechanics.

Let me show you. If you have motion in one dimension, in 1D, you have the equation of motion, motion in one dimension, and there are potential V of x, that this time independent-- a particle moving in one dimension x with under the influence of a potential, V of x. The second-- the dynamical variable is x of t. The dynamical variable. And the equation of motion is-- so let me explain this. This is force equal mass times acceleration. This is mass, this is acceleration, the second derivative of the position, and V force is minus the derivative of the potential evaluated at the position.

You know, derivatives of potentials-- if you think of a potential, the derivative of the potential is here positive, and you know if you have a mass here, it tends to go to the left, so the force is on the left, so it's minus. So V prime is the derivative of V with respect to its argument. And the problem is that while this, taking derivatives, is a linear operation. If you take two derivatives of a sum of things, you take two derivatives of the first plus two derivatives of the second.

But yes, its-- this side is linear, but this side may not be linear. Because a potential can be arbitrary. And that the reverse-- so suppose the potential is cubic in x. V of x goes like x cubed. Then the derivative of V goes like x squared, and x squared is not a linear function. So this, Newton's equation, is not a linear equation. And therefore, it's complicated to solve. Very complicated to solve.

So finally, we can get to our case, quantum mechanics. So in quantum mechanics, what do we have? Quantum mechanics is linear. First, you need an equation, and whose equation is it? Schrodinger's equation, 1925. He writes an equation for the dynamical variable, and the dynamical variable is something called the wave function. This wave function can depend on t-- depends on time-- and it may depend on other things as well. And he describes the dynamics of the quantum system as it evolved in time. There is the wave function, and you have an equation for this wave function.

And what is the equation for this wave function? It's a universal equation-- i hbar partial derivative with respect to time of psi is equal to H hat of psi, where H hat is called the Hamiltonian and it's a linear operator. That's why I had to explain a little bit what the linear operator is. This is the general structure of the Schrodinger equation-- time derivative and the linear operator. So if you wish to write the Schrodinger equation as an L psi equals 0, then L psi would be defined i hbar del/del t of psi minus H hat psi. Then this is the Schrodinger equation. This equation here is Schrodinger's equation.

And as you can see, it's a linear equation. You can check it, check that L is a linear operator. Therefore, it is naturally linear, you can see, because you do it differently, because the derivative with respect to time is a linear operation. If you have the ddt of a number of times a function, the number goes out, you differentiate the function. ddt of the sum of two functions, you differentiate the first, you differentiate. So this is linear and H we said is linear, so L is going to be linear and the Schrodinger equation is going to be a linear equation, and therefore, you're going to have the great advantage that any time you find solutions, you can scale them, you can add them, you can put them together, combine them in superpositions, and find new solutions.

So in that sense, it's remarkable that quantum mechanics is simpler than classical mechanics. And in fact, you will see throughout this semester how the mathematics and the things that we do are simpler in quantum mechanics, or more elegant, more beautiful, more coherent, it's simpler and very nice.

OK, i is the square root of minus 1, is the imaginary unit, and that's what we're going to talk next on the necessity of complex numbers. hbar, yes, it's a number. It shows up in quantum mechanics early on. It it's called Planck's constant and it began when Planck tried to fit the black value spectrum and he found the need to put a constant in there, and then later, Einstein figured out that it was very relevant, so yes, it's a number.

For any physical system that you have, you will have a wave function and you will have a Hamiltonian, and the Hamiltonian is for you to invent or for you to discover. So if you have a particle moving on a line, the wave function will depend on time and on x. If you have a particle moving in three dimensions, it will depend on x vector. It may depend on other things as well or it maybe, like, one particle has several wave functions and that happens when you have a particle with spin.

So in general, always time, sometimes position, there may be cases where it doesn't depend on position. You think of an electron at some point in space and it's fixed-- you lock it there and you want understand the physics of that electron locked into place, and then position is not relevant. So what it does with its spin is relevant and then you may need more than one wave function-- what is one describing the spin up and one describing the spin down?

So it was funny that Schrodinger wrote this equation and when asked, so what is the wave function? He said, I don't know. No physical interpretation for the wave function was obvious for the people that invented quantum mechanics. It took a few months until Max Born said it has to do with probabilities, and that's what we're going to get next.

So our next point is the necessity of complex numbers in quantum mechanics.