PROFESSOR: Suppose you define now, one state called phi 1 as a dagger acting on phi 0. You could not define any interesting state with a acting on phi 0 because a kills phi 0, so you try phi 0 like this. Now you could ask, OK, what energy does it have? Is it an energy eigenstate? Well it is an energy eigenstate if it's a number eigenstate. And we can see if it's a number eigenstate by acting with the number operator. So N phi 1 is equal to N a dagger phi 0. OK.

Here comes trick. Maybe it's too much to even call it a trick, number one. This thing you look at it and you say, I want to sort of simplify this, learn something about it. If this is supposed to be an eigenstate of N hat, I have to make it happen somehow.

Now n hat kills phi 0. So if I would have a term a dagger times N hat near phi 0, it would be 0. So I claim, and this is a step that I want you to be able to do also quickly, that I can replace this by the commutator of these two operators. The product is replaced by the commutators. Why? Aren't products simpler than commutators? No. We have formulas for commutators. And products are, in general, more complicated.

And why is this correct? And you say, well, it is correct because this has two terms. The term I want minus a dagger N hat. But the term a dagger N hat is 0 because N hat kills phi 0. So I can do that because this is N dagger a hat, which is what I had, minus a hat dagger N on phi 0. And this term is 0. So you would have put a 2 here or a 3 here, or any number even. But the right one to put is the commutators. So that's this.

And now this commutator is already known. That's why we computed it. It's just a dagger. So this is a dagger phi 0, and that's what we call phi 1. So N hat on phi 1 is phi 1. N hat has eigenvalue 1 on phi 1. So N is equal to 1. That's the eigenvalue. It is an eigenstate. It is an energy eigenstate. In fact how much energy, E, is h bar omega times N, which is 1, plus 1/2, which is 3/2 h bar omega?

And look what this is. This is the reason this is called a creation operator. Because by acting on the ground state, what people sometimes call the vacuum, the lowest energy state, the vacuum is called the lowest energy state, by acting on the vacuum you get a state. I mean, you've created a state, therefore.

How is this concretely done? Remember you had phi 0 of x, what it is, and a dagger over there is x minus ip over m omega. So this is x minus-- or minus h bar over m omega d dx. So you can act on it. It may be a little messy. But that's it. It's a very closed form expression.

Now, phi 0 was defined, the ground state such that it's a normalized state. This means the integral of phi 0 multiplied with phi 0 over x is 1. That's how we had the ground state.

You could ask, if I've defined phi 1 this way, is simply normalized? So I'll try it. And now you could say, oh, this is going to be a nightmare. Normalizing phi 0 is difficult. Now I have to act with a dagger, which means act with x, take derivatives. It's going to grow twice as big. Then I'm going to have to square it and integrate it. It looks very bad.

The good thing is those with these a's and a daggers, you have to compute anything, pretty much. See how we do it. I want to know how much is phi 1 with phi 1. Is it 1? And it's normalized or not? Then I say, look, phi 1 is a dagger phi 0, a dagger phi 0. So far so good. But I just know things about phi 0. So let's clear up one phi 0. At least I can move the a dagger as an a. So this is phi 0 a a dagger phi 0.

Can I finish the computation in this line? Yes, I think we can. Phi 0. a with a dagger, same story as before. a would kill phi 0. So you can replace that by a commutator. Commutator of a with a dagger phi 0.

But the commutator of a with a dagger is 1, so this is phi 0 phi 0 and it's equal to 1. Yes, it is properly normalized. So that's the nice thing about these a's and a daggers. Just start moving them around. You have to get practice. Where should you move it? Where should you put it? When you replay something by a commutator, when you don't. It's a matter of practice. There's no other way. You have to do a lot of these commutators to get a feeling of how they work and what you're supposed to do.

Let's do another state. Let's try to do phi 2. I'll put a prime because I'm not sure this is going to work out exactly right. And this time, I'll put an a dagger a dagger on the vacuum. Two a daggers, two creation operators on the vacuum.

And now I want to see if this is an energy eigenstate. Well, this is a dagger squared on the vacuum. So let's ask, is N hat-- is phi 2 prime an eigenstate of N hat? Well I would have N hat on a dagger squared on phi 0.

Again, by now you know, I should replace this by a commutator because N hat kills the phi 0, so N hat with a dagger squared phi 0. And that commutator has been done. It's two times a hat dagger squared, two times a dagger squared on phi 0, which is 2 phi 2. That's what we call the state phi 2 prime. I'm sorry.

So again, it is an energy eigenstate. Is it normalized? Well, let's try it. Phi 2 prime phi 2 prime is equal to a dagger a dagger. Let me not put the hats. I'm getting tired of them. a dagger a dagger phi 0.

Now I move all of them. This a dagger becomes an a, the next a dagger becomes an a here. So this is phi 0 a a a dagger a dagger phi 0. Wow, this looks a little more complicated. Because we don't want to calculate that thing, really. We definitely don't want to start writing x and p's.

But, you know, you decide. Take it one at a time. This a is here and wants to act on this thing. And then this other a will, but let's just concentrate on the first a that wants to act.

a would kill phi 0, so we can replace this whole thing by a commutator. So this is phi 0. The first a is still there, but the second, we'll replace it by the commutator, this commutator.

I've replaced this product, the product of a times this thing, by the commutator of those two operators. And then I say, oh look, you've done that. a with a dagger to the k is k a dagger k minus 1. So I'll write it here. This will be a factor of 2 phi 0 a. And this is supposed to be now a dagger to one power less, so it's just a dagger phi 0.

So this is supposed to be 2a dagger. So that's what I did. And again, this a wants to act on phi 0 and it's just blocked by a dagger, but you can replace it by a commutator. a with a dagger phi 0. And this is therefore a 1, so this whole result is a 2.

So this phi 2 prime, yes, it is the next excited state. Two creation operators on the ground state. Energy and eigenvalues too. You had N equal zero eigenvalue for the ground state 1 for phi 1, 2 for phi 2 prime. But it's not properly normalized. Well, if the normalization gives you 2, then you should define phi 2 as 1 over the square root of 2 a dagger a dagger on phi 0. And that's proper.

So it's time to go general. The n-th excited state, we claim is given by an a dagger a dagger, n of them, acting on phi 0 with a coefficient 1 over square root of-- we might think it's n, but it's actually, you can't tell at this far-- this one is n factorial. That's what you need.

That is the state. And what is the number of this state? What is the number eigenvalue on phi n? Well, it is 1 over square root of n factorial. The number acting on the a daggers, the n of them, phi 0. You can replace by the commutator, which then is 2 times already. So it's N commutator with a dagger to the little n phi 0 times 1 over square root of n.

And how much is this commutator? Over there. This is N times a dagger to the n phi 0. So between these three factors, you're still getting n phi to the n. So the number for this state is little n. It is an energy eigenstate. The N eigenvalue is little n. And the energy is h bar omega. The eigenvalue of N hat, which is little n plus 1/2. So it is the energy eigenstate of number little n. This is the definition.

And the last thing you may want to check is the normalization. Let me almost check it here. No, I will check it. Let's say I think this is a full derivation. Phi n with phi n would be two factors of those, so I would have 1 over n factorial a dagger a dagger, n of them on phi 0, a dagger a dagger, n of them again on phi 0. So then that's equal to 1 over n factorial phi 0 a a, lots of a's, n of them, n a daggers, phi 0, like that. That's what it is.

We had to move all the a daggers that were acting on the left input of the integral, or the inner product, all the way to the right. And that's it.

So now comes this step. And I think you can see why it's working. Think of moving the first a all the way here. Well, you can replace the first a with a commutator. But that a with lots of a daggers, with n a daggers, would give you a factor of n, with n a daggers will give you a factor of n times one a dagger less.

So to move the first a, there are n a daggers and you get one factor of n from this a. But for the next a, there's now n minus 1 a daggers, so this time you get a factor of n minus 1 when you move it. From the next one, there's going to be n minus 2 a daggers, so n minus 2. All of them all the way up to one, cancels this n factorial, and that's equal to 1.