PROFESSOR: Let me demonstrate now with plain doing the integral that, really, the shape of this wave is moving with that velocity. So in order to do that, I basically have to do the integral.

And of course, if it's a general integral, I cannot do it. So I have to figure out enough about the integral. So here it is. We have psi of x and t. It's integral dk phi of k e to the ikx minus omega of kt.

OK. It's useful for us to look at this wave at time equals 0 so that we later compare it with the result of the integral. So phi psi at time equals 0 is just dk phi of k e to the ikx.

Only thing you know is that phi has peaked around k0. You don't know more than that. But that's psi of x and time equals 0. Let's look at it later. So we have this thing here. And I cannot do the integral unless I do some approximations.

And I will approximate omega. Omega of k, since we're anyway going to integrate around k0, let's do a Taylor series. It's omega of k0 plus k minus k0, the derivative of omega with respect to k at k0 plus order k minus k0 squared.

So let's--

do this here. So if I've expanded omega as a function of k, which is the only reasonable thing to do. k's near k0 are the only ones that contribute. So omega of k may be an arbitrary function, but it has a Taylor expansion.

And certainly, you've noted that you get back derivative that somehow is part of the answer, so that's certainly a bonus. So now we have to plug this into the integral. And this requires a little bit of vision because it suddenly seems it's going to get very messy.

But if you look at it for a few seconds, you can see what's going on. So psi of x and t, so far, dk phi of k e to the ikx.

So far so good. I'll split the exponential so as to have this thing separate. Let's do this. e to the minus i. I should put omega of k times t. So I'll begin. Omega of k0 times t. That's the first factor.

e to the minus i, the second factor. k--

k d omega dk.

k0 times t. And the third factor is this one with the k0. e to the minus-- it should be e to the plus. i k not d omega dk.

k0 t. Plus order--

higher up. So e to the negligible--

negligible until you need to figure out distortion of wave patterns. We're going to see the wave pattern move. If you want to see the distortion, you have to keep that [INAUDIBLE]. We'll do that in a week from now.

This is the integral. And then, you probably need to think a second. And you say, look. There's lots of things making it look like a difficult integral, but it's not as difficult as it looks. First, I would say, this factor--

doesn't depend on k. It's omega evaluated at k0. So this factor is just confusing. It's not-- doesn't belong in the integral. This factor, too.

k0 is not a function of k. d omega dk evaluated at k0 is not a function of k. So this is not really in the integral. This is negligible. This is in the integral because it has a k. And this is in the integral.

So let me put here, e to the minus i omega of k0 t e to the minus-- to the plus i k0 d omega dk--

at k0 t. Looks messy. Not bad. dk. And now I can put phi of k.

e to the i k x minus these two exponentials, d omega dk at k0 times t. And I ignore this. So far so good.

For this kind of wave, we already get a very nice result because look at this thing.

This quantity can be written in terms of the wave function at time equals 0. It's of the same form at 5k integrated with ik and some number that you call x, which has been changed to this.

So to bring in this and to make it a little clearer-- and many times it's useful. If you have a complex number, it's a little hard to see the bump. Because maybe the bump is in the real part and not in the imaginary part, or in the imaginary part and not in the real part.

So take the absolute value, psi of x and t, absolute value. And now you say, ah, that's why. This is a pure phase. The absolute value of a pure phase is that.

So it's just the absolute value of this one quantity, which is the absolute value of psi at x minus d omega dk k0 t comma 0.

So look what you've proven.

The wave function-- the norm of the wave function-- or the wave. The new norm of the wave at any time t looks like the wave looked at time equals 0 but just displaced a distance.

If there was a peak at x equals 0, at time equals 0. If at time equals 0, psi had a peak when x is equal to zero, it will have a peak-- This function, which is the wave function at time equals 0, will have a peak when this thing is 0, the argument.

And that corresponds to x equals to d omega dk times t, showing again that the wave has moved to the right by d omega dk times t. So I've given two presentations, basically, of this very important result about wave packets that we need to understand.