YEN-JIE LEE: So welcome back again to 8.03. Today, my plan is to continue the discussion of the two string system, which we were working really hard last time but we sort of run out of time. So we didn't have time to enjoy what we have done, right? So today we are going to discuss all the outcome of that calculation.

And so we will start to discuss more examples which can be described by the wave equation. Today we are going to talk about another example, which is sound waves. It's a very exciting topic. And afterwards, we will start the discussion about electromagnetic waves.

So this is the wave equation which we have been using, and over the last few lectures, we have been discussing two specials kinds of solutions-- the normal modes, which is actually standing waves in the end, which we identified, and the progressing wave solution, which is very powerful in describing the phenomena which we are familiar with.

Last time, in the end of the lecture, we were discussing about an interesting example, which involves two strings in the system. One essentially in the left-hand side have mass per unit length, rho l equals to rho 1, and right-hand side one is thicker, and therefore, the mass per unit length, rho, is larger, which is called rho 2.

What we did last time is to assume that we have a progressing wave, which essentially going into this-- which essentially first initiated in the left-hand side string and it's going towards the boundary of the two systems-- the more massive one and the less massive one. And see what is going to happen.

And what we actually tried to describe last time is that we actually define incident wave described by this fi function, transmitting wave, ft, and the refractive wave, fr. By using boundary conditions which we described last time we can conclude that there's a fixed relation between the three waves equation-- wave functions.

What we actually concluded that last time is that fr is proportional to fi, which is the incident wave function, by some constant, which is called r. And what is r? As we solved last time, it's V2 minus V1 over V1 plus V2, which are the velocities of the first and second string. And the transmitted wave, ft, is actually also proportional to the incident wave. And the coefficient-- we call it tau-- to describe the amplitude of the transmitted wave.

So what we can do is the variance. So we actually discussed two examples last time, plugging in V1 and V2. And there are two more examples we can actually make use of this equation we obtained last time to discuss what would be the physics outcome of this kind of situation.

So the first example which we can actually discuss is that, OK, now I have this string. Assume, though, they're connected to a wall. So this time we can say, oh, wait, wait, wait, wait a second. There's only one string now, right? But last time we were solving two strings, right? But what I'm doing now is to treat the wall as if it's a string.

But this wall is really massive. Therefore, the rho l, or the mass per unit length, is really large. OK. It goes to infinity. If that's the case, if you're set this idea-- this is still a two string system-- then I can now go ahead and calculate what will be the velocity. The velocity V2 goes to 0.

Then I can go ahead and plug into my equations. So we spend a lot of time in the last lecture to obtain those equations. And we will find that if I have V2 goes to 0, I have r equal to minus 1. And the tau, which is actually related to the amplitude of the transmitted wave, is equal to 0, which you can see from this equation.

What does that mean? That means once we solve that question, we also know what would be the outcome of this experiment, this physical situation. When we have a string attached to a wall and we have an incident wave, as a function of time, what is going to happen afterward is that this wave is going to propagate and hit the wall and get refracted completely. The amplitude ratio is minus 1. Therefore, all the energy is refracted by the wall in this highly idealized situation.

So that's kind of interesting. The second example is also very interesting. So if you have a string attached to a massive ring, and this ring can go up and down freely without any friction, you can again say, no, no, no, this is again a single string system, right? But what I'm going to argue now is that, OK, there's another string which is so light mass per unit length is close to 0. So it is actually in the air.

If I do that, what is going to happen? The rho l is going to go to the limit of 0, because the right-hand side you almost cannot see it. And the V2, which is the velocity of the transmitted wave, goes to infinity. If that's the case, you can then again, plugging into this equation we obtained, and then you conclude that r would be equal to 1 and tau will be equal to 2.

So what does that mean? This means that if you have this end, which is the open end, attached to a ring which can move up and down, then you get that refraction. The amplitude of the refractive wave doesn't change, because i is equal to 1. So it's still in the positive direction we defined. But now, it goes backward.

Again, all the energy is actually refracted, as you can see from this equation. But the curious case is that-- the strangest thing is that the tau is equal to 2. That's kind of strange, right? Tau is equal to 2. What does that mean? That means you are going to predict a transmitted wave with amplitude exactly the two times the incident wave, and it's going to be propagating in the right-hand side and the speed goes to infinity.

What does that mean? Does that mean the energy is not conserved? Have we found the cure of the energy crisis? Because now-- I can actually take all those energy. I can design this thing, and then this thing will bounce around all over the place. And that is going to emit energy. Oh my god, we solve all the problem. You should be really excited about it, right? No?

But unfortunately, rho l goes to 0. So there's actually nothing oscillating out of this system. So therefore, there is no additional energy radiated out of this system. Too bad. Go back to work.

All right. So that's actually what we discussed last time, and I hope that complete the loop. And today, before we actually move to sound wave, I would like to talk about, very briefly, harmonic progressing waves.

So now, we can see that harmonic progressing wave looks really beautiful, as you can see here. And it can be described by a cosine kx minus omega t plus 5. 5 is actually the face. And you can always write it in different forms. And since we have learned how to describe in general the progressing wave, this is just to remind you that, OK, there's no proper notion to describe a harmonic progressing wave.

So since we have learned about waves, which involve oscillation in the transverse direction. So basically, we always say, OK, things are oscillating up and down in the case of string. Before I start, though, the sound wave, there's a different kind of wave which we can also see very often in the daily life. This is called longitudinal waves.

For example, I can have a spring wave, and I can actually-- imagine I have a spring wave. And I can do this. I oscillate in the horizontal direction. Then that can produce displacement with respect to the equilibrium position. And this kind of behavior is like a density wave. We call it longitudinal waves.

This is exactly what is happening with sound wave. So what is actually sound wave? Essentially, a collection or motion of air molecules. And they are actually oscillating back and forth. And we may use that to extend energy all over the place. And today we are going to discuss the sound wave.

And by the way, just for simplicity, because drawing all those dots really take a lot of time. So what we sometimes do is that, OK, we can now draw the pressure, the amplitude of the pressure, or the amplitude of the displacement of individual molecules in the discussion as a function of time. So if we draw the amplitude as a function of time or as a function of location, then it looks exactly the same as what we discussed before for the transverse waves.

So just some clarification. It's not like the molecules are going up and down. They are going back and forth, and it's just a matter presenting that these are.

So this is actually an example of a travelling wave in the longitudinal direction. And you can see that it is actually the density which is actually changing as a function of time. And as you can see, it's actually traveling at a fixed speed and going in the right direction of the blackboard.

So those being said, we can actually get started with a concrete example. So I would like to discuss with you now a system, which is like you have a tube, with cross section area, A. So A is actually the area of the cross section. And I can now wonder-- now the physics question I'm asking is, what would be the-- what would be the behavior of the air inside this tube?

So before I go ahead and solve this problem, I need to define and give you some more information about this tube and also the condition or the environment this tube is living in. So the first information I would like to give you is that the pressure, the room pressure, is actually P0 in this example. So the P0 is actually the room pressure. And I can now define coordinates is the x direction is actually in the horizontal direction pointing to the right-hand side. And now, I can actually try to describe a small unit volume inside the tube by location x. And the width of this volume, I call it delta x.

And if I go ahead and prepare this system and at time, t, something is happening to this system-- so now, you need length. You need volume, I was discussing. This get displaced with respect to the equilibrium position. So that means, assuming that something happened at the t equal to t, the left inside edge of the volume is shifted toward a positive direction, which is described by wave function psi x. And the position of the right-hand side edge of this volume is shifted to side x plus delta x. So something happened to this system.

We can also say that-- we can also describe this system, the pressure of this system, by P function. P of x is actually equal to P0, which is actually room pressure-- P0 is the baseline room pressure-- plus some kind of displacement in pressure, psi P.

So now we describe the pressure acting on the left inside edge of the small unit volume, and the right-hand side, you can also do the same thing. P of x plus delta x will be equal to P0 plus psi P, describing the displacement or how offset the pressure is as a function of x but now evaluated at x plus delta x.

So once we have all of those elements defined-- these are essentially just a copy of what I have in the slide and those are a reminder here-- now, we can actually calculate the motion of all the molecules in this volume, because I have pressure, I have displacement. The displacement is described by psi, end position is described by psi, the change in pressure is described by psi P. And now I can go ahead and apply, for example, Newton's law. Then I can calculate what would be the acceleration for all the molecules inside this volume.

But wait a second. That sounds all great, but I don't know yet how to relate pressure and the volume, because pressure is actually expressed by psi P and the volume is related to psi. I need to know is actually the relation between pressure and the displacement or pressure between psi so that I can make progress.

So that this actually the main discussion which I would like to do in this lecture. So given those information, I can now calculate what is actually the change in this little volume. So I can calculate the change in volume, which is described by delta V. But delta V can be actually calculated by a, which is actually the area of the cross section, times psi x plus delta xt minus psi xt. So basically, just calculate how much the boundary is actually displaced.

And if we always take very small amplitude approximation, then basically this expression is roughly equal to A partial psi partial x times delta x, where the delta x is really very small. So a very small volume I was talking about. And I can also calculate the pressure.

What is the pressure difference? The pressure difference is between the pressure acting in the left-hand side edge and the pressure which is acting on the right-hand side edge. So I can now calculate pressure difference, delta P. Delta P would be minus psi P x plus delta x t plus psi P x t. So basically, one is essentially the pressure pushing the body in the right-hand side. The other one essentially pushing it in the left-hand side direction.

Again, I can take very small delta x approximation. And basically, what you are going to get is minus partial psi P partial x delta x.

So we have prepared all of those information about volume and the pressure. As I mentioned before, the big question which we would like to ask is, how do I relate pressure and the volume so that I can make progress? If I can relate pressure and volume, then I can know what is the relation between psi P and the psi. Then I can ask you to make use of Newton's Law. Then I can calculate the resulting equation of motion.

So there's two possible interesting scenarios which we can relate temperature-- so sorry, relate pressure and the volume. The first one was proposed by Newton. Newton said that, OK, this is an interesting phenomena. In my opinion, although you actually displaced this volume-- make the displacement for those molecules in the tube-- but because the heat was conducted from one region to the other region, all those regions are connected to each other.

And the speed of this heat transfer is so fast. It's really fast, like instant. This heat is actually transferred from one direction to the other-- one position to the other position. Therefore, over the course of this evolution, the temperature should be unchanged. No matter what you do to the air inside the tube, the temperature should be unchanged, because Newton thinks that heat should be-- the speed of the heat distribution is really, really fast. Much faster than all those vibration happening in the tube.

If that is the case-- that is the case-- then that means we can use ideal gas law. P times V is equal to nRT. I hope that you have learned this before in 8.01 and 8.02. If that's the case, that means-- so all those things are constant, because we assume that temperature is unchanged. Therefore, the right-hand side is essentially a constant. Therefore, P times V would be some kind of constant. The V would be proportional to 1/P. So that essentially is the first idea, which is coming from Newton, in order to relate pressure and the volume.

The second idea is coming from Laplace. Laplace says, OK, he has a different opinion on this matter. He think that this essentially is an adiabatic process. What does that mean? That means the heat flow from the compressed region to the other region is really negligible, because the oscillation is really fast and the speed of the transfer of the heat is really slow compared to the time scale of the oscillation. Therefore, in Laplace's opinion, he thinks that the whole process is adiabatic process.

If that's the case, which I will show you later, that means you have this relation between pressure and the volume. P times V to the gamma. Gamma essentially related to the decrease of freedom of the molecule, which we will discuss later in the class. This would be equal to constant.

So the very interesting thing of this lecture is that we are going to be able to test which one is correct. You will be able to see if Newton win or Laplace win. So as I mentioned before, one is assuming the heat transfer, the speed of the heat propagation, is really, really much larger than the speed of oscillation. The other viewpoint from Laplace is that the heat flow is actually really negligible compared to the oscillation we are talking about here.

And now, as usual, I would like to have a vote now. How many of you support Newton's idea? 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11-- so 15 of you vote for Newton. How many of you saying Laplace is correct? How about the others? OK, very good.

So we have a majority of you support the idea of Laplace, and some of you actually support Newton. And we are going to see what is going to happen in the lecture today. So let's go ahead and apply these two ideas.

So PV gamma equal to constant. So in the case if ideal gas law, gamma is equal to 1. Therefore, I just have to work on these function of form. And then also later we will figure out what is gamma all together. So that's consider only small vibration. Small vibration means-- small vibration means that I have psi P, which essentially, from this definition, psi P is the change in pressure with respect to the room pressure, P0.

So psi P, assuming that's much, much smaller than P0. And also, I assume that the changing volume, delta V, which I calculated there, is much, much smaller than P0, which is essentially the original volume of this little area-- original volume of this area I was working here.

All right. So that's the two assumption. Before I change the position of the boundary, which essentially is the upper graph, if I change anything, I have P0 V0 gamma. This is equal to some constant, C. And gamma can be equal to 1 so that you have ideal gas law.

After the vibration initially happened, after the wall's initially displaced from the equilibrium position, what I'm going to get is-- I will have P plus delta P times V0 plus delta V to the gamma. This is equal to C.

So based on those, actually I have already calculated delta P and delta V. So in this case, this delta P should be-- OK, this is actually not the delta P I was talking about there, so I should change it to delta P. Because that's, essentially, the difference between the resulting pressure and the original pressure. It's not the difference between the left-hand side pressure and the right-hand side pressure, which essentially is showing there.

So this delta P, which I have already defined, essentially called psi P. Therefore, I will derive this to be P plus psi P. And I can now also copy these. You are going to get V0 plus delta V to the gamma. And this is equal to C.

So as I mentioned before, I'm considering small vibration. Delta V is much, much smaller than V0. Therefore, this expression can be written as P-- sorry, this should be P0. I'm making some mistake here. This expression should be written as P0 plus psi P. V0 to the gamma. 1 plus gamma delta V over V0, because delta V is much, much smarter than V0. Can everybody follow?

So here I will already take small vibration approximation. And basically, I can now rewrite this thing. I just expend all those terms. Basically, I call this equation number two. Equation number two will become P0 V0 gamma plus gamma delta V V0 gamma to the minus 1 P0 plus psi P V0 to the gamma plus gamma delta V psi P V0 to the gamma minus 1.

So there's no magic. Essentially, it's just expanding these terms. Then, basically, you are going to get four terms. And basically, if you do write down the equation number two especially, that is essentially what you are going to get.

So we are making progress, and we would like to simplify things. And you can quickly identify the hardest term, P0 V0 to the gamma. I know what is the value of that, right? That essentially is the original situation, and that is essentially equal to C.

Let's take a look at also this term. This term is proportional to what? Proportional to delta V, which is a very small quantity, and proportional to psi P, which is another very small quantity. Therefore, taking a small vibration approximation, I would just simply ignore this term. Is everybody following? All right.

So this term, this original term, is equal to C based on this expression. So we start from the system before the vibration happened. After the vibration happened there's a change in pressure, there's a change in delta V. But if you multiply PV to the gamma, this expression is still equal to C, some kind of constant.

And then, now I do small vibration approximation, and I drop the term which is actually proportional to delta V times psi P. And basically what I get is that C is equal to C plus gamma delta P V0 gamma minus 1 P0 plus psi P V0 to the gamma.

This term, these two constants cancel. And now, I can actually move one of the terms to the left-hand side. Then basically, what I am going to get is psi P V0 to the gamma would be equal to minus gamma delta V gamma 0 gamma to the-- V0 to the gamma minus 1 times P0.

We can't immediately cancel V0 to the gamma. Therefore, what are we going to get? I'm getting psi P would be equal to minus gamma P0 over V0 delta V. Everybody following?

So this is essentially the expression. And we also know why essentially is delta V. Based on this expression, delta V is essentially A partial psi partial x delta x, if we look at the upper board, which we actually just derived a moment ago. Therefore, I can write, replace delta V by that expression. Then basically what I get is psi P would be equal to minus gamma P0 A delta x divided by V0 partial psi partial x. I'm just plugging in the expression for delta V to that equation.

A lot of mathematics, but all of them should be pretty straightforward. You don't actually have to copy because all of them are in the lecture note. OK? All right.

So here, you can see I can, again, simplify this expression. A is actually the cross section of the tube, and delta x is the width in the x direction. So A times delta x is just V0. Oh, very good! I get this very simple expression, minus gamma P0 partial psi partial x.

So we have achieved our goal to simplify the expression and to find the relation between psi P and the psi. That's actually what originally we were hoping to do, and we have achieved that. And I call it equation number three here. Don't forget what is psi and psi P. Psi P is the amount of change in pressure, and the psi is the amount of displacement of the wall-- of the molecule in the volume.

So now, I'm really close to my solution, because now I can now calculate the force acting on this little volume, because now I know what is the pressure. So what is the F total? The F total is essentially delta P, which is the difference in pressure from the left-hand side end compared to the right-hand side end. So that's what we calculated before. Now this, A partial psi P partial x delta x.

We also know what would be the mass. We know that the little mass in this volume, delta m, will be equal to rho, which is the density of the air, times A, which is the cross section, times delta x. That will give you the little area, V0. So rho times A times delta x will be your delta m.

We are almost there. I have the m, I have the force, what kind of law do I need to use to get my equation of motion?

AUDIENCE: Newton's law.

YEN-JIE LEE: Newton's law, right? Newton's law. So F is equal to m times a, right? So therefore, I can now calculate and essentially I can now plug in rho times A times delta x. What is A? A is essentially psi double-dot. That's essentially describing the displacement with respect to the equilibrium position. And now I know this is equal to force, which is A times delta x partial psi P partial x.

And you can see that both ends you have a delta x. So I can cancel that. Both ends you have an A, so now I can cancel that. Then basically you get rho psi double-prime. This is equal to partial psi P partial x.

From the beginning we're talking about the relation between psi P, the displacement in pressure, and psi, how much the molecules are displaced. And then we have the solution here. If you assume the relation which was given by Newton or by Laplace, basically, you can conclude that this would be equal to gamma P0 partial square psi P partial x square.

All right. And I can now put all the constants to the right-hand side. Basically, what you get is psi double-prime. So here it should be partial square psi partial x square, because I replaced psi P by psi. And I must miss one-- I must miss one negative sign somewhere.

AUDIENCE: Over there.

YEN-JIE LEE: Where?

AUDIENCE: [INAUDIBLE]

YEN-JIE LEE: Oh, this essentially-- there's a minus sign there, right?

AUDIENCE: [INAUDIBLE] On the left side.

YEN-JIE LEE: On the left-hand side. Yeah. That's right.

AUDIENCE: Oh, no. There should be A. That's wrong.

YEN-JIE LEE: Yes, you are right. And the minus sign should belong there. So that actually-- sorry for that. So there should be a minus sign here. And there should be a minus sign here. And after I plug in equation number three, then I get psi double-prime equal to gamma P0 over rho partial square psi partial x square. Any other problems you find? Not yet? OK.

So look at this equation. Oh my god! What is this equation?

AUDIENCE: Wave.

YEN-JIE LEE: Wave equation. Again. Again. Wave equation. You can say that, huh, I'm not surprised, because this system is used so many times. I have learned this so many times, but I am still surprised that this is so identical to the physics which we have been studying for the strings for over the few lectures. So that's very nice.

And now, the question we have an answer is that, OK, what essentially is gamma? What is essentially gamma? So gamma, in the case of the adiabatic process, gamma is actually equals to alpha plus 1 minus alpha. And alpha is related to the number of degrees of freedom.

So if you haven't done this before, I have a concrete proof of the adiabatic process. And this is actually coming from the first law of thermodynamics. And basically, you will be able to conclude that gamma will be equal to alpha plus 1 divided by alpha. The value of alpha-- the value you get for alpha is related to how many degrees of freedom you can actually have in this system.

For example, if I have a system which is made of atomic gas-- so there's only one atom in a molecule-- and basically you have 3 degrees of freedom. So you can move this thing in the horizontal direction. You can move this atom upside down or back and forth. So there are three degrees of freedom. And if you calculate alpha, that will give you 3/2. And basically, if you calculate gamma according to the equation you are going to get 1.67.

On the other hand, if you have atomic gas, that means you have more degrees of freedom. So basically, you have not only the translation of degrees of freedom-- the three ones which are identical to the atomic gas-- you can also have two rotational degrees of freedom. So you can have these two atoms rotating like this, you can have that rotating like that.

The trickiest thing is this one vibration degree of freedom, as you learned from the couple equations before. But this one, very trickily, is not excited at all at low temperature. You have to go to really, really super high temperature so that this actually contribute to the overall degrees of freedom. So therefore, you have a total of available degrees of freedom of 5. And if you calculate the gamma, you basically will get 1.4.

So let's now calculate what will be the resulting speed of light. Sorry, no. Not speed of light. The speed of sound So if the temperature remain unchanged, if you take this equation here, this is a wave equation. Therefore, I know how to calculate the speed of sound.

The speed of sound will be equal to-- P will be equal to the square root of gamma P0 divided by rho.

So I have figured out the rho and the room pressure for you. So the P0 will be 10 to 5 kilogram ms squared, and the rho will be equal to 1.2. Rho is actually the density of the air. It's essentially 1.2 kilogram per meter cubed.

If I have a gamma equal to 1, which is the case for ideal gas law temperature unchanged, if I calculate the resulting speed of sound you are going to get something like 389 meters per second. So that's the prediction for Newton.

And the second case, if we have-- if we are believing what Laplace actually said, the heat flow is really, really negligible compared with the speed of oscillation, then we have, as we discussed last slide, gamma would be equal to 1.4. Therefore, you would be able to calculate the resulting speed of sound, and that is actually 342.

So those are the predictions. And what I'm going to do now is to really demonstrate that we can actually measure the speed of sound in front of you. So the first thing which I will need to do is to switch so that you can see the camera. And now, I have a set up here.

Basically, this set up is like the following. So basically, very similar to the setup we have here. But at one end, we actually have a speaker which produced sound wave. So this is essentially what we have.

This is the tube, and we have, one end, there's a speaker attached to here, produce a sound wave. And basically, the amplitude will look like this. So basically, this will create some kind of standing wave inside the tube.

And I have another device, which is actually a microphone. A microphone is connected to this scope, which shows you the amplitude of the-- basically, the amplitude measured by this microphone. And you see that if I move this, as a function of position you see that the am is changing. It's getting smaller when it is actually hitting the note here, because here there's almost no oscillation in the air. Therefore, you will measure a very small signal at that position.

And if you continue to move, then you can see that, aha, I move away from the note, therefore I see some kind of maxima. Then, I see that this amplitude is dropping again. If I continue-- if I continue, then say that, aha, again, this amplitude is increasing to a very large value. Then it decreases to a minima around the note.

So what I going to do now is to measure the distance between notes. And since the sound waves, which I actually put into the system, have a frequency of-- let me see. The frequency I put in-- the frequency I put in is actually 1 kilohertz.

With the location of the note, I can know what will be, what? What will be the wavelengths of the sound waves. So therefore, I can now measure the distance between those three notes. Then I would be able to measure the wavelengths. Then I would be able to know who is correct-- if Newton is correct or Laplace is correct.

So let's do that. So let me find the first minima. So the first minima is around 64. 64 centimeter. And you can see that. Say stop when you see that it's reaching the minima again. Stop. OK, very good.

So this is actually the first note, the location of the first note, and I was trying to find the next note so that I can actually readout the wavelengths. Now, this will increase again. And reach-- stop? Is that a stop sign? OK, very good.

All right. So I get the value, which is actually 30 centimeter. So now I can calculate what would be the lambda. The lambda is actually 64 minus 30. Then, what I get is 34 centimeters. And if I calculate the velocity of the sound wave, then basically I have F times lambda, and that will give you-- this is actually equal to 0.34 centimeter. So that would give you 340 meters per second.

Oh my god. This is so close to the prediction of Laplace. First of all, this is amazing. Why? Because with this looks really crappy thing, I can measure speed of sound. Secondly, ooh, the measurement is really great. It match within 1%. You guys did a good job of stopping me. Very nice. And finally, very unfortunately, people who voted for Newton is wrong.

So what happened is the following. What happened is that Newton actually assumed that the speed of propagation of the heat is really fast, but actually that's not true. Because, for example, I'm standing here and heating up the air. In the last few lecture, I even heat up the air by some kind of fire in front of you. But you don't feel the heat, right?

So the heat propagation is really not really, really fast compared to the speed of vibration. The vibration is really quick, because it's really vibrating up and down 1,000 times per second. So that means what is actually much more reasonable is to describe this process is adiabatic process.

So we will take a five minute break here so that we can take questions, and then we'll come back at the 39.

So welcome back, everybody. So we can see that from the last-- so from the discussion we had before the break, we see that the sound wave can be described by something which we are now very familiar with-- the wave equation. And also, we know what is the speed of the sound, which, based on this wave equation, the speed of sound is actually equal to square root of gamma P0 over rho.

So gamma is actually obtained from this discussion of how many degrees of freedom we have. So the first case we discussed is, if you have a single atom of which you make your air, then basically the gamma is actually higher. On the other hand, if you have diatomic gas, then the gamma is actually slightly smaller. It's 1.4.

So what would happen if I change the air in my lung to monatomic atom? So what is going to happen is that the speed of sound is going to be increased. The speed of sound will increase. And I have a fixed sized of lung. I didn't increase the size of my lung. Therefore, the frequency of my sound will what? Will increase.

So how about we do that experiment and see if it works? So here, I have a balloon here, which is full of helium. Let me see if it works. I'm not sure if it will work, but let's see. Fingers crossed.

We'll see what happens.

Now I'm going to do a measure of operation to replace all the air in my lung by this. Does my sound change?

[LAUGHTER]

No? Didn't work. Let me do that again. I speak more aggressive.

[LAUGHTER]

Did you hear any difference? No.

(HIGH PITCHED) Any difference? Works? Works now? Very good. Maybe we should use some-- maybe we should use that sound to go over all the lecture, right? It's a very dangerous experiment, because you are replacing all the air in your lung. So you may choke. Fortunately, I survived this experiment and hope you enjoy.

[APPLAUSE]

What happened is the following. So basically-- basically, the gamma becomes large. Therefore, the speed of sound in my lung becomes large. Therefore, the frequency of my sound increased and you hear some really strange sound. OK, very good.

So before the end, I would like-- poor Newton. Before the end, I would like to discuss with you something which I hope I would not see again in the exam but I saw before. So if I create a progressing wave in a single-- in a closed end, open end tube, this progressing wave is going to be propagating at the speed of what? The speed of what?

AUDIENCE: Sound.

YEN-JIE LEE: Sound, yes. It's going to be propagating at the speed of sound, and you would reach the boundary. The question is, will we see this? Would this progressing wave just simply leak out of the tube. How many of you think that's going to happen? I hope I will never see that again in the exam.

This would never happen. Why? That means you will have a super narrow collimated progressing wave going straight out of a tube, and that will not actually match the boundary conditions at the end of the tube. So that means, basically, first of all, there was no refraction. That means that all the energy is transferred outside of the world.

And according to what we discussed before, what you would expect is that, OK, now you suddenly change to environment which you have really very large volume. Therefore, you it will be very difficult to change the pressure outside of the tube. Because what you are actually connected to is a reserve of infinite number of molecules outside. It's going to be really hard to change the pressure.

Therefore, apparently this behavior doesn't match the boundary condition. And therefore, what you should expect is something like this, which I can show you here. So in this case, you have both side opened. What is going to happen is that at the boundary-- actually, it's like the case of hitting a wall, because outside of the tube you have really, really large volume, huge amount of air out of it. Therefore, it's like hitting a wall. The amplitude of the progressing wave changes sine and goes back through the tube.

And of course, this system is actually not perfect. Therefore, there can be some leaking out-- some energy leak out of the tube, which essentially must be happening, because we can actually roll the tube and we can hear the sound. That is because some of the sound wave actually leaks out of the tube. And this process will go over and over again. And this progressing wave is going to be going back and forth, like that.

So I hope that after this demonstration everybody will expect that, OK, this will be not-- the result will be like this progressing wave is going to be reflected because of the boundary condition and also change sine in terms of amplitude.

So what we have learned today-- so it's already close to the end. We have learned example of a longitudinal wave And basically, longitudinal wave is actually in the form of density wave in the example which we covered today. And the mathematical description of the sound wave is going to be almost identical to what we have learned from the string case, which we actually discussed last time.

There are two boundary conditions which I would like to briefly discuss before we end the lecture today. So in the case of open end, as we discussed before, we can have a system which contains a closed end and an open end. What will be the boundary condition for a closed end?

So the closed end have a wall here. Therefore, when you have your molecule, the air molecule oscillating back and forth, when they are actually close to the wall, they cannot vibrate. Why? Because it's hitting the wall. It cannot vibrate so that actually, the boundary condition at the closed end, where you have a wall closing the tube, is psi equal to 0.

And on the other hand, if you have an open end-- if you have an open end, that means outside of the tube the pressure is equal to what? It's equal to P0. The room pressure. And you have so many stuff there. Therefore, it's not possible to actually change the pressure dramatically at the edge of the open end.

Therefore, what will be the condition? Psi P. Psi P is again the displacement with respect to the room. Pressure will be equal to 0. Based on what we actually have learned from this expression-- sorry for that, my finger slipped. From this expression, psi P is equal to minus gamma P0 d psi dx So psi P is proportional to partial psi partial x. Therefore, this boundary condition actually translates to partial psi partial x equal to 0.

So this issue looks really familiar to you, because in terms of psi, if you forget about this system, what those boundary conditions mean to you is exactly the same as you have some kind of a wall in the left-hand side and it's connected to a string, and the right-hand side of the string is connected to a massless ring which can actually move up and down.

These two systems, if you actually don't look at the detail-- only look at the wave functions and the boundary conditions-- they are identical. So that's actually the first lesson we learn from here. So when I talk about sound wave or when you think about sound wave problem, there's nothing to be afraid of anymore, because that's actually the same as what we have learned with wall and a string system. That's the first thing we learned.

Secondly, that only works when I write my wave function in the form of psi. So now I can actually get the first normal mode would be like this if I plot psi as a function of x. The second normal mode-- doesn't surprise you-- will look like this, et cetera, et cetera. If I plot psi-- if I plot psi as a function of x.

On the other hand, we also know that psi P is proportional to d psi partial psi partial x. Therefore, you can also plot psi P as a function of x. Then what you are going to get is something like this. In the closed end, the psi P is actually reaching the maxima, because it's got the wall. Therefore, it can actually produce pressure on top of the wall. But you cannot move the position of all of those molecules in front of the wall. Therefore, that makes sense.

You will see exactly in the opposite direction, if you plot the amplitude as a function of x, you see a picture which is almost like flipped. Of course, you can also do the same thing for the second one. And basically, what you are going to get is something like. The second normal mode, et cetera, et cetera.

So be careful about the matching between the boundary condition obtained from the tube and string-wall system. They are identical. Open corresponds responds to open, closed corresponds to closed when you express your equation of motion in terms of psi. On the other hand, if you change that to psi P, then the relation is actually flipped.

Thank you very much, and I hope you enjoyed the lecture today. And I will see you next week.