PROFESSOR: OK, welcome back, everybody, to 8.03. Today we are going to continue the discussion of wave equation starting from last lecture. So what have we learned last time? As a reminder, we have started to study the behavior of a wave equation. We understood basic behavior of the wave equation, and also are trying to solve the general solution of the wave equation. The first thing which we learned, as usual, is the normal modes. What are the normal modes in the case of wave equation over continuous translations in metric system.

What we found last time is that they are standing waves. And of course, as usual, the full solution, which is a general description of this system, is the superposition of infinite number of normal modes. And that means one can understand this kind of system systematically the using Fourier series. So this is just a reminder. So what we have done is that basically we start with infinite number of coupled oscillators. And we make the space between those massive objects in the system smaller and smaller until it become continuous, right? And a very interesting thing happened that automatically already give you wave equations, OK? Which is as you're shown here.

Last time, as I mentioned, we discussed the normal modes, which are standing waves. Those are the first few normal modes, and those are the functional form of the normal modes, which are standing waves. So the structure just looks like this. So you have A m, which is the amplitude, sin k m x plus alpha m, which can be determined by boundary conditions. And the sin omega m t plus beta m, that means all the points in the system are oscillating at the same frequency, omega m, and as the same phase, which is beta m.

And those are based on the wave equation. If you plug this solution back into the equation, you will find that omega m is actually not a free parameter. It's actually proportional to k m, which is the wave number of m's normal mode. And this constant of v p, we will find out today, this is actually the speed of the wave in the case of traveling wave. And if you look at the individual normal modes and make product of those normal mode as a function of time, you can see from here there are six different normal modes. And they are like a sinusoidal shape in terms of amplitude as a functional position on the string. And there you can see that if you distort the string more, then you get a higher oscillation frequency, as you see from m equal to 1 to m equals to 6 case.

So as I mentioned today, we will talk about another interesting kind of solution, which is progressing wave solution, OK? Why is this solution exciting? Not just because this actually matches what we actually already learned about waves, right? And this solution should enable us to send, for example, energy from one point to another point. This is actually what I am doing now, right? I'm sending energy from my mouth to your ear so that you can hear what I have been talking about, right? About 8.03, right? So that's really cool. And we will try to understand how actually we can get this solution out of this strange wave equation, OK?

So what is showing here is the normal mode. And we are going to talk about a second kind of solution, which is the progressing wave solution. And this solution have this form, psi (x, t) will be equal to some kind of function, f. And this is actually a function of x minus v p times t. This is actually a general form of the progressing wave. And f function is some kind of well-behaved function of your choice. OK, so the first thing which I would like to do is to show that this functional form is actually the solution of the wave equation, right? So fairly straightforward. We can actually go ahead and plug them into this equation.

And before that, I will define tau to be x minus v p times t, OK? So in order to prepare for plugging in this functional form to a wave equation, I would calculate using chain law. Partial f partial x will be equal to partial f, partial tau. Partial tau, partial x. And this will give you partial f, partial tau times, in this case, partial tau, partial x will give you 1, OK? And that will give you f prime tau, OK? Therefore we can go ahead and calculate as well partial square f, partial x squared, OK? That will give you f double prime tau. So this is actually the first set of equation I need in order to describe the right side of the wave equation.

The other equation which I need in the preparation for plugging in the whole thing into the wave equation is to calculate partial f, partial t. And according to chain law, partial f, partial t is equal to partial f, partial tau, partial tau, partial t. In this case, f is actually a function of x minus v p t, and tau is defined as x minus v p t. Therefore you can actually immediately conclude that these would be equal to minus v p, partial f, partial tau. And now it is actually equal to minus v p f prime. Similarly, you can calculate partial square of f, partial t squared, and that would give you a v p squared f prime. Oh, f double prime because we did the double differential.

OK, all right. So from the first equation and second equation, which I have on the board, we can actually plug that into the wave equation. And what I'm going to get is partial square f, partial t square. That would be equal to v p square, partial square f, partial x square. So that is actually exactly the wave equation, which we showed from the beginning. So that means this functional form satisfy the wave equation. And I didn't even specify what is actually f function. f function is some kind of well-behaved function. And it can have all kinds of different shape, which we will discuss later.

But that is actually pretty encouraging. And that means if you try to distort the string and if this shape is actually propagating at-- have the functional form of x minus--

If you have any function which you have a functional form of f of x minus Vpt-- no matter what kind of shape it is, this shape is going to be-- first of all, this function is a solution to the wave equation.

Secondly, we will show you that this shape is going to be propagating at the speed of Vp. And the shape will not change. It will stay like that forever according to the wave equation.

There's another function of form which can also be the functional form of the progressing wave. We usually write it as f KX plus or minus omega t. And in this case if omega is actually Vp times k, which is actually already required from the discussion of normal modes. Then this equation-- this kind of functional form is also the solution of wave equation.

And of course you can actually go ahead and prove that probably after the lecture. The proof will be very similar to what we have done here.

So now, actually try to understand what does this function mean? So I have a function which is f of x minus Vp times t. Where x is actually the position of the-- on the string. And the t is actually the time, which I go ahead and check this function.

And for example, if I can give you an example-- a function, for example, I can make it like a triangular shape like this. And this is actually plotted as a function of Tau. So f of Tau is actually giving you the information of the shape of the progressing wave.

So let's discuss if I write f of x minus Vt, what does that mean? OK, first of all, I can take t equal to 0. What will what happen is that a lot instants of time, t equal to 0, x is equal to Tau.

That means at t equal to 0, the shape of the string-- well, it looks like this-- like this function.

Now, I would like to know how this string is going to evolve as a function of time. So that's actually all we care, right?

And that means I'm going to increase t to a larger value. So suppose originally, I have a t equal to 0, I'm sitting on this point. I sample the f at this position. If I increase t-- if I increase t from 0 to a larger value where I will start to sample, will I move to a left-hand side, or a right-hand side? Anybody can help me. Because this functional form is actually x minus Vt. What will happen if I increase t? Tau will increase or decrease?

AUDIENCE: Decrease.

PROFESSOR: Decrease.

AUDIENCE: Yeah. Move to the left.

PROFESSOR: Very good. So decrease, right? So that means at the fixed x, I am going to sample this point. What does that mean? That means originally, if I plot everything in terms of x-- originally, it have this shape.

Now, if I increase t, what is going to happen is that originally I was sampling this shape here. And now I am sampling the shape here.

That means if t equal to t prime, which is larger than t, this shape first of all is unchanged. Secondly, it's actually moving. The shape looks like as if it's moving in the horizontal direction.

And the direction of this movement is in the positive x direction. If I'd write my progressing wave solution in a functional form of f of x minus Vt-- this should be Vt here, sorry for that. Any questions?

So look at what we have done. First of all, we have proved that f of x minus Vpt, this function of form is a solution to the wave equation. It's a solution.

Secondly, we also discussed the property of this functional form. So that's essentially describing a shape. And the whole shape is going to move as if it's moving as a function of time and to the positive x direction.

So let me ask you another question. So what will happen if I write the solution in the form of f of x plus Vt? Anybody can help me with the direction of the propagation of the wave.

AUDIENCE: Go to the left.

PROFESSOR: Yeah. Go to the left-hand side of the board. That means if I have another expression. I'm using the same f function which is defined here. In this case, if I write f of x plus Vt, that means the shape will be moving in the negative x direction. It's symmetric.

Of course you can also discuss what will happen if you take this functional form and you are going to get exactly the same conclusion.

So right now I have been talking about moving shape. So what is actually really moving? Professor Lee, you just told us before that every point on the string can only move up and down. Now, you are talking about something moving in the positive x and negative x direction. What does that mean?

So that mean-- take y example. So if I have a Gaussian pulse, and I write this thing in the form of x minus Vt. At t equal to 0, this shape looks like this. In the next moment, if I increase time to t equal to 1, what is going to happen is that this shape move toward the positive x direction.

And of course 0's are the equilibrium position of this waves-- of the string. And what is happening is it's like this-- so basically, all the points on the string are really working together to produce this shifting-- this progress Gaussian wave.

So what is happening is that if I focus on this point, this point will go down. And this point will go down, go down, go down, until I touch this point. So basically, what is happening is like this-- all the points are only moving up and down horizontally. But they all move in a manner such that if you look at just the shape of the amplitude-- the amplitude is a function of x-- it looks as if the amplitude-- the shape is actually moving toward positive x direction. Moving toward the right-hand side of the board.

So what is actually moving? What is moving is actually all the point like mass on the string-- they are only moving up and done. But they are moving together so nicely such that it looks as if the whole shape is actually shifting toward the positive x direction.

Any questions so far? So I hope that's straight forward enough. And I would like to discuss with you a interesting situation. So we have learned that, OK, I can have, for example, triangular pulse.

And I can have this triangular pulse moving in the positive x direction. And that we will also find out that the speed of the propagation is actually Vp because that's actually if you increase t and that's actually the speed of movement when you sample the shape and the f of Tau.

So therefore, we can conclude that the speed of this triangular pulse is going to be Vp. If I have another triangular pulse starting from the right-hand side end of the string, they have exactly the same shape, exactly the same amplitude.

So of course, according to what we actually wrote there, they are going to move forever. And at some point they will actually meet each other. And what is going to happen is that you will have some pulse, which is actually two times of the shape at some point. Because of the linearity of the wave equation. So that's actually pretty straight forward.

However, if I consider another case, which is like this. So I have two progressing waves. One is actually going in the right-hand side direction. The other one is going to the left-hand side direction.

They have exactly the same shape, but they have-- the amplitude is actually taking the minus sign. So they actually are exactly-- they have exactly the same amplitude, but pointing to a different direction. One is actually pointing upward. The other one is actually pointing downward.

So at some point these two waves is going to overlap each other. When they overlap each other, what is going to happen? It's like this-- they are going to overlap each other. That means the amplitude will be cancelling each other. Then from the experiment you will see something like this.

So now, this is the question I would like to ask you. What will happen next? The first possibility is that they cancel. Completely, they disappear. The second possibility is that, OK, they pass each other. The third possibility is that, OK, it depends on the mood of the string. Maybe something interesting is popping out. Maybe it decide to produce two circular waves. Get creative, right? Creative.

OK, everybody have to vote. OK? How many of you think that you will cancel and disappear. Anybody? Nobody? Really? So you can see that here, nothings there. Right? Why didn't you think that will be canceled? OK, nobody think that will cancel. Very good. Maybe we are all wrong, right?

[LAUGHTER]

Second, they'll pass each other. How many of you think so? Very good. Finally, how many of you think that will be, oh, no, it depends on the mood of the string. Get creative. One, two, three-- thank you for the support.

[LAUGHTER]

There are four people. OK.

So let's discuss these three situations carefully. So the first situation, if they cancel exactly, which sounds logical because if you look at this string how could this string remember what happened before? How could it remember?

Therefore, shouldn't answer number one be a logical choice? The catch is, OK, if they cancel then that means energy is not conserved. So somehow the energy I put in-- I work really hard to shake the string, use my energy. And it disappear. Oh my god, disappear.

[LAUGHTER]

Then the energy is sad. The second one is, OK, I believe in energy conservation. So they will pass each other. But that means the string have memory because right now there's nothing there. What is going on?

OK, since most of you think that is actually what is happening, can some of you explain to me how this string actually remember what happened before? Anybody can help me.

AUDIENCE: Maybe the two light forms reflect off of each other. Bounce off of each other or something.

PROFESSOR: Yeah, they balance each other, but how is this different from a stationary string at rest? Of course, I mean at some point it looks identical, right? But there's something which is different between this one and that one.

AUDIENCE: There's no [INAUDIBLE]. No [INAUDIBLE] in the string. [INAUDIBLE].

PROFESSOR: Very good point. This one, which is actually unperturbed, has zero velocity. And this one, no. It actually have a got velocity. Actually, this string is already or starting to-- it's already ready to move down. And this part of the string is already ready to move up. So that is actually how the string can remember what happened before.

It remembered it by the velocity. So what is actually not plotted here is a trick. It's actually the velocity. The velocity is already nonzero compared to this situation. And what is going to happen is that afterward you will produce two corresponding triangular pulse, continue and then pass each other.

Finally, the third condition, creative. That may not happen because all the memory is still there in the form of kinetic energy.

So we can actually go ahead and do a small demonstration here. So let's focus on the right-hand side part of this setup. So this is actually the Bell Lab machine we had before. So now, what I'm going to do is now I'm going to create a square pulse-- positive square pulse from the lab inside. And the negative pulse in the right-hand side and see what is going to happen.

No, not like this. Stop. Stop. OK. All right. Let's do it. You see? They pass each other. And the shape actually continues. So let's do that again. They cancel at some point, but they do pass each other and continue. And there are some refractions, et cetera, which we are going to discuss afterward.

Let's do that again. You see? At some point, they cancel. But the positive pulse continue traveling to your left-hand side. And then the negative pulse travel to your right-hand side. Continue, please.

So based on the experiment most of you actually were correct. The answer is number two. And I would like to show you a few more examples based on my little simulation.

So first of all, I would like to show you a triangular pulse. They pass each other. And you can see that they pass each other, and the shape is actually changing as a function of time. And actually, afterward, they continue, and they keep the same shape based on this computer simulation.

Another interesting thing to notice is that if you focus on the point at x equal to 0, you will see that at this point actually never change amplitude. Because those two pulse are really symmetric. One is positive, the other one is negative.

As usual, we can actually change the shape of the pulse. For example, I can changed it to circular shape and see what will happen. Oh.

[LAUGHTER]

And again, the position at x equal to 0 is unchanged. Let's take a look at that again. It really does something really funny. It looks like, voom. And then you can see that it is actually the velocity of the individual component of the string. Which it remember though in the original shape. So it can see the velocity by eye looks different from what you see it before in the first example.

And finally, as usual, we have the MIT waves.

[LAUGHTER]

And it does really, really crazy things. And the amazing thing is that the string have such a good memory. It really remember what is going to happen before they touch each other.

So what is going to happen to these two MIT waves? They are going to be propagating forever. Cannot stop until the edge of the universe. Maybe they dig out of the universe, but not my problem anymore.

So we talk about the energy stored in the string and et cetera. So how about we go ahead and calculate the kinetic energy and the potential energy.

So the first part is the kinetic energy. Only one is actually half mv square. So if I consider a small segment on the string, which have a width of delta x. And I can now calculate delta m.

If I assume this string have a mass per unit length Rho L, and the string tension t. If that's actually given to you when we set up the experiment, then we can actually calculate the mass of this small portion of the string.

Then delta m, the mass, will be equal to Rho L times dx. Because Rho L is the mass per unit length. Therefore, what is actually the kinetic energy is becoming pretty straight forward. It's the integration over the whole string. Integration over the whole string is 1/2-- based on this equation-- Rho L, dx, and times v.

But what is actually v here? v is actually the velocity of individual point-like mass on the string. And we actually already talked about that. The velocity of individual mass is actually only in the y direction. And the position of individual mass is described by the function Psi.

Therefore, what is actually velocity? Velocity is actually partial Psi, partial t. So that is actually giving you the velocity of individual mass on the string. And then if you square that, that is actually giving you the total kinetic energy-- is in this functional form.

Let's also discuss what is actually the potential energy. The potential energy as you remember delta W, the work, is equal to F times delta S, the displacement. F is the force, and the delta S is the displacement.

So originally, before we actually perturb and make some displacement with respect to equilibrium position-- this string have originally-- if I look at this small part of the string I join in this region. This looks like this. This is delta x, and it has a constant string tension t.

Now, I can actually introduce some displacement. And what is going to happen is, look, it's going to look like this. This string is actually a little bit stretched. And this is actually the original delta x. The width of this little segment.

And this direction is actually a small change in the y direction, which is actually showing us delta Psi. And of course, we can calculate the length. The length of this string and that will give you square root of delta x square plus delta Psi square.

We can now go ahead and calculate the delta W. So delta W will be equal to F, which is the force, times delta S. We know in the force-- the magnitude of the force is what? Is the string tension. So therefore, I put T here.

And delta S, what is delta S? Is how much I stretch this string. So this is actually the difference between the resulting length and the original length, delta x.

So that is actually giving you the delta S. So that means I can write it in this functional form. dx square plus d Psi square minus dx.

I can of course take delta x out of this square root thing, and basically I get delta x, square root of 1 plus d Psi dx square minus dx.

Remember what we have been discussing until now, we were always discussing small amplitude-- or small vibration. Therefore, that means I can use a small angle approximation. That means delta Psi is going to be very, very small with respect to delta x.

So that means the first turn will be roughly delta x 1 plus d Psi dx squared 1/2 because you have a square root of that, plus higher order turn.

And of course we assume that delta Psi is actually much smaller than delta x. Therefore, we ignore all those higher order terms.

So if we actually replace this expression back into the original equation, you will see that the first turn, 1 cancel with this minus dx turn. This actually cancel that. They actually cancel.

Therefore, I can calculate dW will be equal to T times delta x, times 1/2 d Psi dx square. Therefore, what will be the total potential energy. The total potential energy will be in the equation of the work dW over the whole range from-- of the system. And basically, you can actually write it down as 1/2 T Psi. Partial Psi, partial x square dx.

All right, so we can actually understand and calculate the kinetic energy and the potential energy. So before we take a break, let's take a short example to check if we understand what we have learned so far.

So for example, if I have a function Psi xt, and that is actually equal to 1 over 1 plus x minus 3t to the fourth. It's a crazy function. If I assume that I can do a very precise thing, manipulate this string so that I produce a wave function of this functional form. 1 over 1 plus x minus 3t to the fourth.

Can somebody tell me what is actually going to be the velocity of the wave? Can anybody tell me?

The first thing which you can do is to express this crazy function in a functional form of fx minus Vpt, right? And the Vp is actually the speed of the wave, right? So anybody know what is actually-- yes?

AUDIENCE: Three.

PROFESSOR: Yeah, the three because the whole function can be written as f x minus 3t. Therefore, the velocity Vp will be 3. Of course if you are not sure, you can actually calculate Vp square by the ratio of partial square Psi, partial t square, and partial square Psi, partial x square. And that will give you of course the Vp square, according to that wave equation.

All right, so we will take a five minute break from now. And during the break I will try to return the exam to you. So we will come back at 24-- 12:24.

So welcome back, everybody. So we will continue the discussion of traveling wave. So we have the very interesting discussion of two waves that are canceling each other. And somehow the string have a way to remember what happened before, which is actually the velocity of each individual point on the string as a function of x-- that instance of time.

So let's actually take a look at this example. So make use of what we have learned so far. As we see here there is a triangular shape, which I create in the lab. And this triangular shape is actually there and it's stationary. It's not moving. The strings are at rest, but have a triangular shape, which I setup there.

So based on what we have learned so far-- we have learned normal modes, we have learned about traveling wave. I believe before we learned this class, the first reaction to you is to do what? What kind of decomposition.

AUDIENCE: [INAUDIBLE].

PROFESSOR: Fourier decomposition. So what you are going to do is, OK, very good. I have this shape. So I do a Fourier decomposition and I have infinite number of terms. And I am going to evolve infinite number of term as a function of time and see what will happen to the system. So that's actually what you would do before we learned traveling wave.

What I would like to say today is that if I really prepare this string at rest, stationary, at t equal to 0-- in contrast to what I just said before, brute force measure, which I used computer to decompose it and evolve all the infinite number of terms. What we could do is that I can show you that this situation is a superposition of two traveling waves.

Then the question becomes super simple. So instead of doing a brute force calculation using computer-- decompose it to infinite number of normal modes-- what I can actually show you is that, OK, if I have a g function, which is equal to f x plus Vpt plus f x minus Vpt.

So these three function is superposition of two traveling waves. The shape is described by f function. And one of them is traveling to the right-hand side. The other one is actually traveling to a left-hand side.

If I assume that the superposition of these two traveling wave is g then I can now calculate the velocity, partial g, partial t. And that will give you Vp f prime minus-- right, because here it's actually x minus Vp-- so I got minus Vp out of it, f prime.

The first term, if I do this partial differentiation, then basically I get the positive Vp out of it. And then the second term I get minus Vp out of it. And these two terms cancel exactly.

What does that mean? That means all the points-- this g is actually a function of x and t-- on the point at t equal to 0 have initial d velocity equal to 0.

So in other word, if I have any random kind of shape-- in this case is a triangular shape-- I can always decompose this stationary shape into two traveling wave.

One is actually traveling in the positive direction. The other one is traveling in the negative direction. So that means what is happening? This is equal to a superposition of two traveling waves. If I assume the height of this mountain to be h then I need to have h over 2 as the height for the individual traveling waves.

One is actually traveling to the right-hand side. The other one is actually traveling to he left-hand side of the board. So based on this trick, actually we can see that I don't need to do infinite number of terms anymore.

I don't need to do a Fourier decomposition and get really crazy and take forever to write the code on your computer. And maybe there are some bug in your code, which is frustrating.

And what we could do is to simply decompose it into two traveling wave. And I can now predict what will happened at time equal to, for example, 5 what is going to happen. So what is going to happen is that you have two triangular shape waves. Each of them actually traveled a distance of 5 times Vp.

So that is actually a very interesting fact. And of course we can see from here if I quickly create a triangular shape, and you will see that it really did become two triangular shape wave. So I can do this. I can do this. You see?

So originally, I'm creating some stationary shape. And I release that.

It does become two traveling waves with amplitude half of the original height-- original displacement. I can also do it in the opposite direction, a positive wave. You see? It does work.

And of course, after the class you can make even more complicated shape if I have many more than two hands. Maybe I can do that, but unfortunately I'm human.

You can see that I can create different slope in the positive and negative h. And it does create two traveling wave. And that's amazing because this is actually looks like just some kind of mathematical trick. And it really match with what we can do experimentally.

So finally, I would like to talk about the last topic of the lecture today, which is to connect two strings together. So suppose I have two strings-- the left-hand side string is actually thinner. It has mass per unit length Rho L, and string tension t.

In the right-hand side you can have a thicker string with mass per unit length 4 times Rho L, and the string tension is as you keep constant t.

Based on what we have learned before, the velocity Vp, is equal to square root of t over Rho L. So that's actually from the last few lectures. So left-hand side you will have V1, which is the velocity of the traveling wave, equal to square root of t over Rho L.

And the right-hand side you will have square root of t over 4 Rho L. And that will give you one half of V1. So what does that mean? This means that if I have a traveling wave in the left-hand side, the speed of the traveling wave will be 2 times the speed of the traveling wave in the right-hand side based on this calculation.

So what I would like to do is the following-- so I would like to ask a question about this system. What will happen if I introduce a displacement and a traveling wave from the left-hand side. And the question is, what is going to happen to this system as a function of time once I actually give this input traveling wave.

And the answer is that this traveling wave is going to pass through the boundary of two systems. And there may be refraction. There may be transmission, et cetera. And that we are actually in the good position to understand this phenomena.

So let's take a look at this situation carefully. So now I define here the position of the boundary is at x equal to 0. And I can now go ahead and write down the conditions, which is-- need to be satisfied in order to connect these two systems properly. Which you actually already see this several times, the boundary condition.

So what are the boundary conditions which I need in order to connect the left-hand side and the right-hand side systems? So the first boundary condition is that the string is continuous. Therefore, if I have some kind of y is actually-- y of x t is describing the displacement of all the time mass on the string in a horizontal direction.

Then that means y the left-hand side evaluated at 0 minus in the slightly left-hand side of the boundary will be equal to YR, which is actually evaluated at the slightly right-hand side of the boundary at x equal to 0.

And the YL is actually the wave function for the left-hand side thinner string. And the YR is actually the wave function, which describe the right-hand side of the string.

So this means that the boundary condition tells us that the string cannot break. It should match carefully so that these two systems are connected to each other properly.

The second condition is that, OK, since this boundary actually have no massive particles left, I can't actually assume that this is massless ring there. Therefore, the slope of the left-hand side, partial YL, partial x, s equal to 0, will have to be equal to the slope at the right-hand side.

If the slope doesn't match between the left-hand side and right-hand side, that means since they have constant tension that means the tension-- the string tension cannot cancel each other. Then the massless ring will be transferred to, for example Mars, in a second. Because it has few infinite amount of acceleration.

And that didn't happen. When I actually tried to actually displace the string or the Bell Labs system, I didn't see crazy things happen.

Therefore, the tension at this, which acting on this massless ring must cancel each other. So that's the second boundary condition we have.

So now, I would like to make some assumption. So first of all I have an input pulse, which is actually coming into this system. Looks like this. And I call it fi, is traveling toward the positive x direction.

So therefore, I can of course write it down as minus k1x plus omega t. So this is actually the incident pulse, I call it fi. And after it pass the boundary-- so I can actually expect that there may be some kind of refraction, which happened at the boundary, fr, I call it fr. And this time this fr is going to be traveling to the negative x direction.

Therefore, I can express this function as fr is a function of plus k1x plus omega t. Finally, there can be also transmission wave. So you get the refraction and there can be some energy, which somehow pass through the boundary. And I call this transmission wave ft, which is actually in a form of minus k2x plus omega t.

And in this case, I assume that the system is actually having a k1 in the left-hand side, and k2 in right-hand side. Which is actually the wave number and the k1 is actually equal to omega over V1. And the k2 is actually equal to omega over V2.

So that is actually the set up. And then also the three traveling waves, which we actually demonstrate the situation. So we can now go ahead and plug those three traveling wave solution into the boundary conditions. And then we will be able to solve their relative amplitudes.

So let's make use of the first boundary condition. So YL is now a superposition of fi and fr. YR will be just the transmission wave, ft.

So now, I can plug this expression back into the equation number one. Then basically, what I get is fi omega t. Originally, it's actually minus k1x plus omega t, but this thing is actually evaluated at x equal to 0.

The wave function has to be continuous between the negative side of 0 and the positive side of 0. Therefore, if I plug in x equal to 0, minus k1 turn disappear. And what is left over is omega t.

And this is the second turn fr, I can write down expressively. You get fr omega t. And then right-hand side of the expression is YR, only have 1 turn, ft And now you are going to get ft omega t.

So now we can also go ahead and plug in this equation to equation number two. What is going to happen is that I do a partial differentiation with respect to x. And the plug in x equal to 0 to the expression. And what I'm going to get is minus k1 f prime i, as a function of omega t, plus k1 fr prime omega t. And this will be equal to minus k2.

In the right-hand side you only have one turn, which is ft. So you are going to have minus k2 ft, the function of omega t.

Any questions so far?

AUDIENCE: [INAUDIBLE].

PROFESSOR: Yes, thank you very much. OK, very good. So we are making progress here. And what I can do now is to do a integration over t for the equation number two. So if I do a integration basically, what I'm going to get is minus k1 over 2 fi omega t.

I do a integration over t, plus-- over omega, sorry-- and the plus K1 over omega fr omega t. And this is actually equal to minus K2 over omega ft omega t.

Based on the equation, which we have before-- K1 over omega is actually 1 over V1. So basically, what we have is actually-- this is actually 1 over V1. This is actually 1 over V1. And this is actually 1 over V2.

So in short while we are going to get in the second equation will become minus V2 fi omega t plus fr omega t. If I multiply both sides by V1 and V2 then I get the minus V2 here.

And this will be equal to-- there should be a minus here because I am taking out minus V2 there. And this will be equal to the right-hand side because I multiply both side by V1 and V2. Actually, I get minus V1 ft omega t.

So what is actually left over is that now I have equation number one, and I have equation number two. Those are just functions of fi, fr, and ft. So that means we can actually easily solve the equation and write everything in terms of fi.

So we can now solve one and two and write in terms of fi, which is actually the incident wave. That's actually what we could do. So if I do that-- if I solved the equation one and two, basically I get fr omega t, will be equal to V2 minus V1 divided by V2 plus V1 times fi omega t.

If you trust me, if I try to solve one and two, and express fr, and ft in terms of fi-- then basically the second thing which I get from this solution is that ft will be equal to 2V2 divided by V1 plus V2, fi omega t.

So look at what we have done. Basically, the first thing which we did is to identify what are the boundary conditions. Under the condition one is the string doesn't break at the boundary. The slope match between the two boundary because you have constant tension.

Then I assume the solution have the functional form of three traveling wave. The incident traveling wave, fi, traveling to the positive direction. The refraction is expressed as fr going to the negative direction. And finally, ft is going to-- is the transmission wave going to the positive direction.

Then I plug those equation in to the boundary condition. And I solve everything, fr and ft, in terms of fi and this is actually what I get. So that's actually in short what I have been doing.

So basically, this expression is actually equal to R time fi, where R is actually V2 minus V1, divided by V1 plus V2. And in this case this is equal to transmission, which I am writing as Tau, times fi, the initial incident wave. And this Tau is equal to 2 times V2 divided by V1 plus V2.

So in this example, V2 is equal to 1 over 2 V1. So I can now plug it in and see what I get. Basically, V2 will be equal to V1 over 2. Then I can evaluate, will be the R and Tau. So the R will be minus 1/3.

It's a negative value. And the Tau will be equal to 2 over 3. So what have we learned from here? So if I create a pulse starting from the one which is actually have-- which is lighter or have smaller Rho L-- smaller mass per unit length-- when it passed through the boundary there will be a refracted wave, which the amplitude will change it's sign.

So what is going to happen is that you will get a reflective wave and the amplitude changes sign. And then there will be a transmitted wave, which is actually going to the positive direction. So this is actually a demonstration we have here. So left-hand side is the system, which I was talking about-- the smaller Rho L system. And right-hand side is the larger Rho L system.

And now I can do the experiment and see what happen. And I connect the two system with this ring, so that they are coupled to each other. I hope you will work.

All right, so now I can create-- oh, I'm in trouble now. One second. Hopefully will-- this is not easy. OK, now I can create a pulse from the left-hand side. Oh, no. That is the pressure. So now I can create a pulse from the left-hand side. And you can see that there is a small pulse actually that pass through the median-- pass through the boundary, but unfortunately this demo is not setup already. Ah, gosh. OK, so we will see what we can get from here.

Now, it works. Very good. So now I can actually create a pulse from the left-hand side, and you can see that it does pass through this boundary if I setup the ring. The ring was falling down somehow during the lecture. And this ring is actually presenting the boundary and connect these two system.

Based on what we predict from the equation-- basically you will see that if I have a positive amplitude passing through the boundary there will be a negative pulse going backward and a positive pulse going through the boundary, which is the transmission wave. And let's see what is going to happen.

You see? It does have a negative pulse going backward. And you do see that there is a pulse, which is actually going through this system. Let's see this again. You see that there's a positive amplitude pulse going through the boundary and that there's a refraction through this-- which is actually going backward in the left-hand side system.

So on the other hand, if I start a traveling wave from your left-hand side, that means V2 is going to be larger than V1. V2 is actually going to be larger than the V1.

So what are we going to get is a positive amplitude refraction and also a positive value transmission wave. And let's see what is going to happen. You see? The refraction is positive this time. And the transmission wave have also positive amplitudes.

Let's take a look at this thing again. A very nice pulse and you can see the refraction because of this mathematics. Interesting thing is that it match with experimental result. And the prediction was that you are going to get the positive amplitude reflective wave, and it does agree with the experimental data.

So this is actually what we have learned. So we have learned traveling wave solution. Energy of a oscillating string, and also the potential kinetic energy. And also we learn how to actually match two media and passings-- how this traveling wave pass through the median, et cetera.

And next time we will talk about more systems described by wave equation. And also dispersion relation, what does that mean, et cetera. Thank you very much, and see you on Thursday.