8.03 Lecture 20

Interference: Superposition of EM waves $\Rightarrow$ Enhance or cancel each other. Consider this physical situation:

$$\vec{E}_1 = A_1 \cos(\omega t - kz + \phi_1)\hat{x}$$
$$\vec{E}_2 = A_2 \cos(\omega t - kz + \phi_2)\hat{x}$$

Where $\vec{E} = \vec{E}_1 + \vec{E}_2$. And recall:

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B} \qquad \vec{B} = \frac{1}{v}\hat{x}\times\vec{E}$$

The Intensity, which is the power transfer per unit area, or the magnitude of the Poynting vector, is $I = |\vec{S}|$

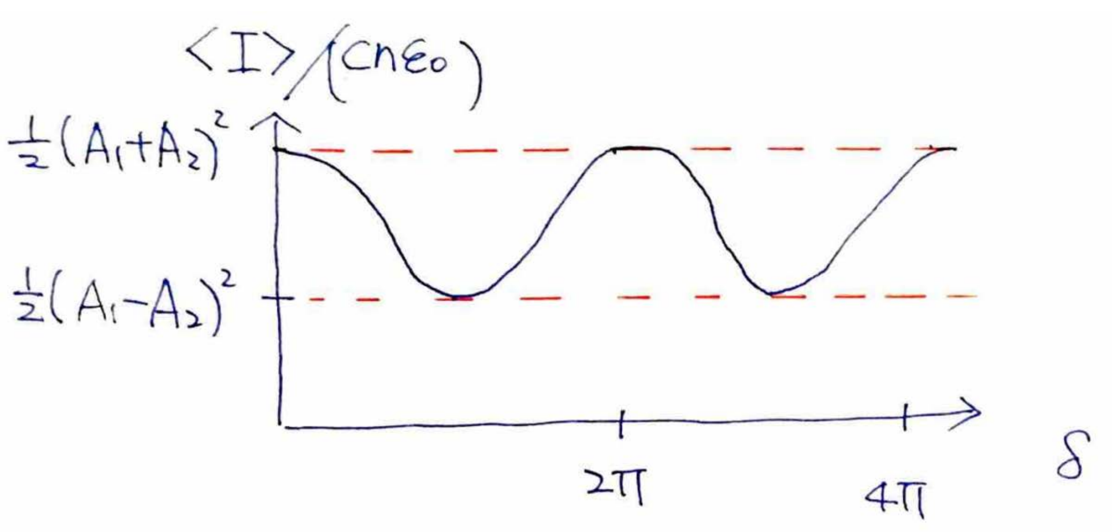
$$|\vec{S}| = \frac{1}{\mu_0}|\vec{E}\times\vec{B}| = \frac{n}{\mu_0 c}|\vec{E}|^2 = cn\epsilon_0|\vec{E}|^2$$

$$\begin{aligned}|\vec{E}|^2 = {} & A_1^2\cos^2(\omega t - kz + \phi_1) + A_2^2\cos^2(\omega t - kz + \phi_2) \\ & + 2A_1A_2\underbrace{\cos(\omega t - kz + \phi_1)\cos(\omega t - kz + \phi_2)}_{\frac{1}{2}(\cos(2\omega t - 2kz + \phi_1 + \phi_2) + \cos(\phi_1 - \phi_2))}\end{aligned}$$

$$\langle I\rangle = \frac{1}{T}\int_0^T I\,dt \qquad \left(\cos^2 x = \frac{1+\cos 2x}{2}\right)$$

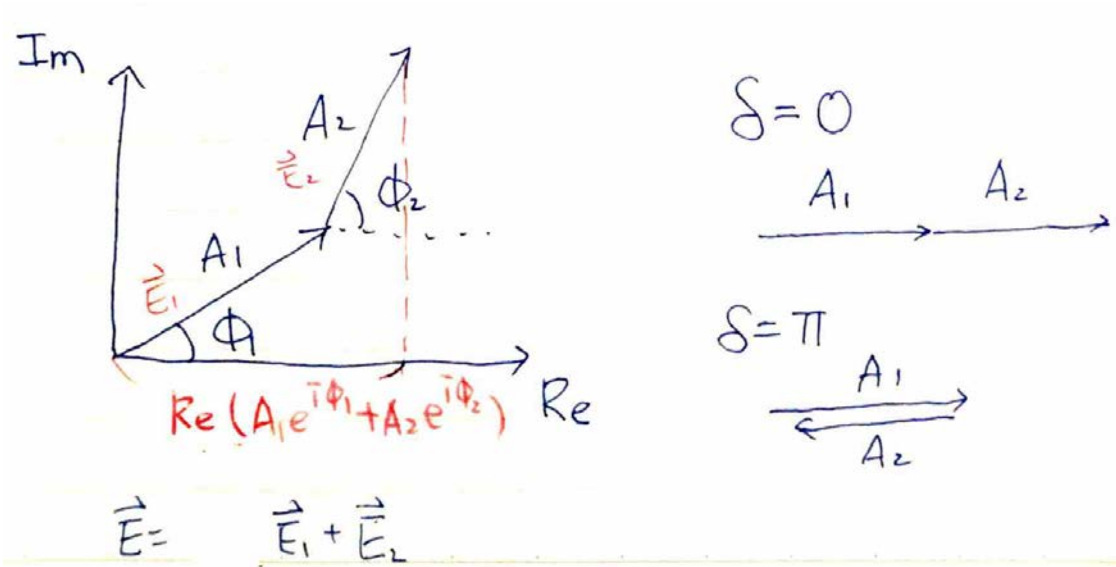
$$|\vec{E}|^2 = cn\epsilon_0\left[\frac{A_1^2}{2} + \frac{A_2^2}{2} + 0 + A_1A_2\cos(\delta)\right]$$

Where we define $\delta \equiv \phi_1 - \phi_2$



If $A_1 = A_2 \Rightarrow$ completely cancel when $\delta = \pi, 3\pi, \cdots$!! How do we understand this? Imaginary plane.

$$\vec{E}_1 = \text{Re}\left[A_1 e^{i\phi_1} e^{i(\omega t - kz)}\right]\hat{x}$$
$$\vec{E}_2 = \text{Re}\left[A_2 e^{i\phi_2} e^{i(\omega t - kz)}\right]\hat{x}$$



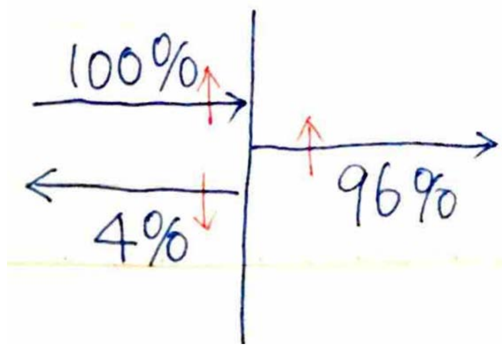
An interesting example: interference involving dielectrics. Last lecture we learned the reflection and transmission coefficients:

$$R = \frac{n_1 - n_2}{n_1 + n_2} \qquad T = \frac{2n_1}{n_1 + n_2}$$

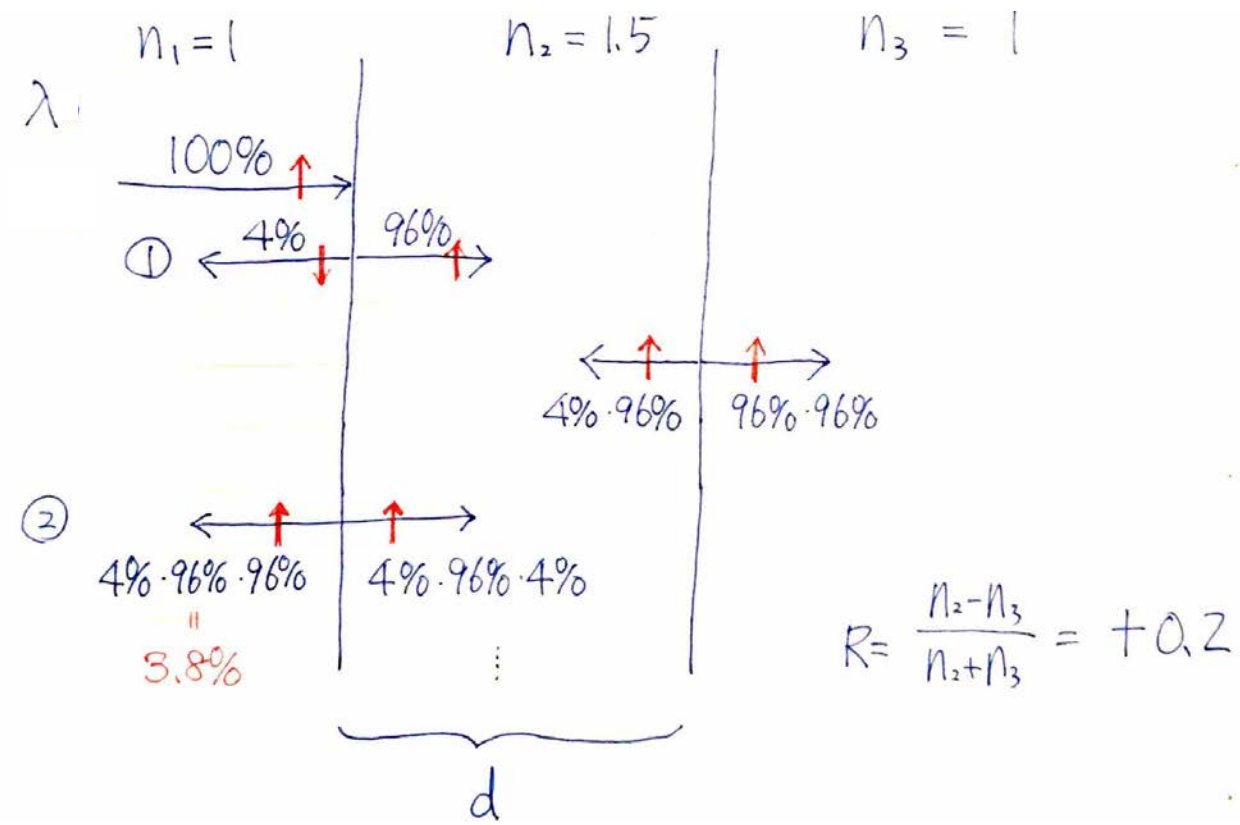
(1.) If $R > 0$ (for example, $n_1 > n_2$)$\Rightarrow$ no flip in amplitude
(2.) If $R < 0$ (for example, $n_1 < n_2$ or $v_1 > v_2$) there is a flip in amplitude and a phase difference of π
Example: $n_1 = 1$ in air and $n_2 = 1.5$ in a soap solution. Recall $I = \frac{cn\epsilon_0}{2}E^2$

$$R = \frac{1 - 1.5}{1 + 1.5} = -0.2 \qquad T = \frac{2}{2.5} = 0.8$$

$$I_R = 0.04\, I_0 \qquad I_T = 1.5 \cdot 0.8^2 I_0 = 0.96\, I_0$$



Now we are in position to understand the soap bubble. Why is it colorful? Translate this physical situation into mathematics. Consider a thin layer of sap water (and simplify by considering normal incidence):



We are looking at the interference between 1 and 2. Question: what is the thickness d that is needed to have constructive interference?
Phase difference: 1-2:

$$\delta = \frac{2d}{\lambda/n_2} 2\pi + \pi$$

Where the first term is from the critical path length difference between 1 and 2 and the second term comes from the change in amplitude.

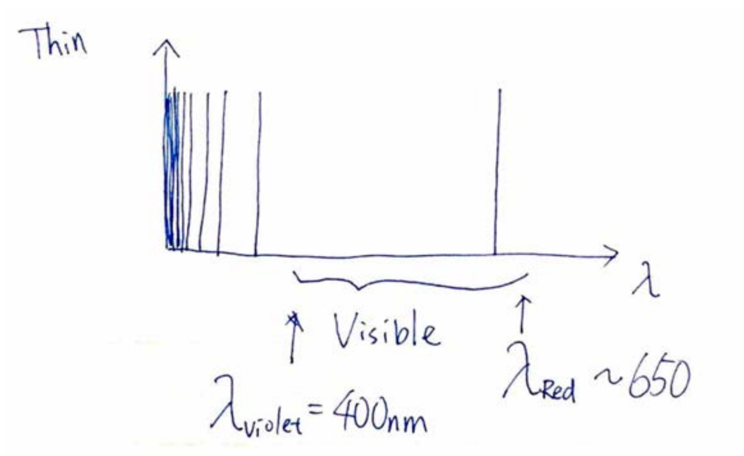
Constructive interference: $\delta = 2N\pi$
Destructive interference: $\delta = (2N+1)\pi$

1. $d \longrightarrow 0$ (very very thin) $\Rightarrow$ Destructive interference
2. Constructive interference:
$$d = \frac{(2N-1)\lambda}{4n_2}$$
3. Destructive interference:
$$d = \frac{2N\lambda}{4n_2} = \frac{N\lambda}{2n_2}$$
4. If we fix the d and change λ
$$\lambda_{\text{Max}} = \frac{4dn_2}{2N-1}$$

Thin layer: If $d \approx 100nm$ then

$$\begin{aligned}\lambda_{\text{Max}} &= \frac{4 \cdot 100nm \cdot 1.5}{2N-1} = \frac{600nm}{2N-1} \\ &= 600nm, 200nm, 120nm, \cdots\end{aligned}$$

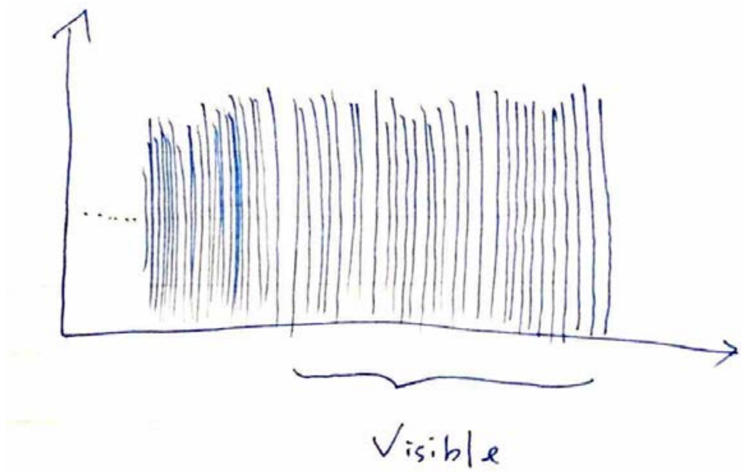
for $N = 1, 2, 3 \cdots$



$\Rightarrow$ We see color in the soap bubble!!
Thick layer: If $d \approx 100\mu m$ $(N = 1)$

$$\begin{aligned}\lambda_{\text{Max}} \quad &= \quad 600\mu m \qquad N = 1 \\ &\vdots \\ &600.6nm \quad N = 500 \\ &599.4nm \quad N = 501 \\ &598.2nm \quad N = 502 \\ &\vdots\end{aligned}$$

A lot of wave lengths in the range of visible light has constructive interference $\Rightarrow$ White in our brain!



We learned:

1. Need $d \approx 100nm \Rightarrow$ colorful soap bubble!
2. No color if thick
3. Color disappears (or bubble becomes transparent) when $d \longrightarrow 0$