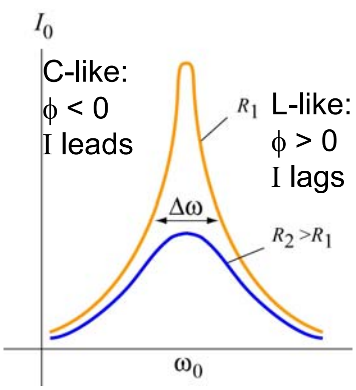
Topics: Driven LRC Circuits
Related Reading:

Course Notes (Liao et al.): Chapter 12
Serway & Jewett: Chapter 33
Giancoli: Chapter 31

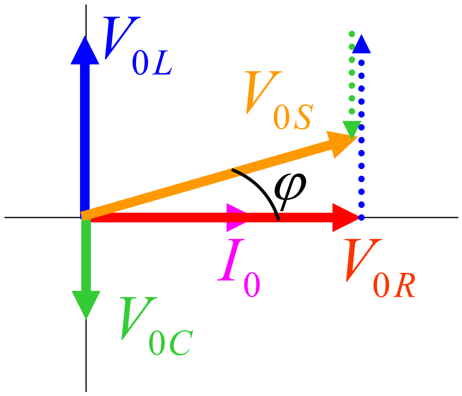
Topic Introduction

Today's problem solving focuses on the driven RLC circuit, which we discussed last class.

Terminology: Resistance, Reactance, Impedance
Before starting I would like to remind you of some terms that we throw around nearly interchangeably, although they aren't. When discussing resistors we talk about their *resistance R*, which gives the relationship between voltage across them and current through them. For capacitors and inductors we do the same, introducing the term *reactance X*. That is, $V_0=I_0X$, just like V=IR. What is the difference? In resistors the current is in phase with the voltage across them. In capacitors and inductors the current is $\pi/2$ out of phase with the voltage across them (current leads in a capacitor, lags in an inductor). This is why I can only write the relationship for the amplitudes $V_0 = I_0X$ and **not** for the time dependent values V=IX. When talking about combinations of resistors, inductors and capacitors, we use the *impedance Z*: $V_0 = I_0Z$. For a general Z the phase is neither 0 (as for R) or $\pi/2$ (as for X).



Resonance
Recall that when you drive an RLC circuit, that the current in the circuit depends on the frequency of the drive. Two typical response curves (I vs. drive ω) are shown at left, showing that at resonance ($\omega = \omega_0$) the current is a maximum, and that as the drive is shifted away from the resonance frequency, the magnitude of the current decreases. In addition to the magnitude of the current, the phase shift between the drive and the current also changes. At low frequencies, the capacitor dominates the circuit (it fills up more readily, meaning it has a higher impedance), so the circuit looks "capacitance-like" – the current leads the drive voltage. At high frequencies the inductor dominates the circuit (the rapid changes means it is fighting hard all the time, and has a high impedance), so the circuit looks "inductor-like" – the current lags the drive voltage. Notice that the resistor has the effect of reducing the overall amplitude of the current, and that its effect is particularly acute on resonance. This is because on resonance the impedance of the circuit is dominated by the resistance, whereas off resonance the impedance is dominated by either capacitance (at low frequencies) or inductance (at high frequencies).



Seeing it Mathematically – Phasors

It turns out that a nice way of looking at these relationships is thru phasor diagrams. A phasor is just a vector whose magnitude is the amplitude of either the voltage or current through a given circuit element and whose angle corresponds to the phase of that voltage or current. In thinking about time dependence of a signal, we allow the phasors to rotate about the origin (in a counterclockwise fashion) with time, and only look at their component along the y-axis. This component oscillates, just like the current and voltages in the circuit, even though the total amplitude of the signal (the length of the vector) stays the same.

We use phasors because they allow us to add voltages across different circuit elements even though those voltages are not in phase with each other (so you can't just add them as numbers). For example, the phasor diagram above illustrates the relationship of voltages in a series LRC circuit. The current *I* is assigned to be at "0 phase" (along the x-axis). The phase of the voltage across the resistor is the same. The voltage across the inductor L leads (is ahead of *I*) and the voltage across the capacitor C lags (is behind *I*). If you add up (using vector arithmetic) the voltages across R, L & C (the red and dashed blue & green lines respectively) you must arrive at the voltage across the power supply. This then gives you a rapid way of understanding the phase between the drive (the power supply voltage V_S) and the response (the current) – here labeled ϕ.

Power

Power dissipation in AC circuits is very similar to power dissipation in DC circuits – only the resistors dissipate any power. The big difference is that now the power dissipated, like everything else, oscillates in time. We thus discuss the idea of *average* power dissipation. To average a function that oscillates in time, we integrate it over a period of the oscillation, and divide by that period: $<P>=\frac{1}{T}\int_0^T P(t)\,dt$ (if you don't see why this is the case, draw some arbitrary function and ask yourself what the average height is – it's the area under the curve divided by the length). Conveniently, the average of $\sin^2(\omega t)$ (or $\cos^2(\omega t)$) is ½. Thus although the *instantaneous* power dissipated by a resistor is $P(t)=I(t)^2 R$, the *average* power is given by $<P>=\frac{1}{2}I_0^2 R = I_{rms}^2 R$, where "RMS" stands for "root mean square" (the square root of the time average of the function squared).

Important Equations

Impedance of R, L, C: $R = R$ (in phase), $X_C = \frac{1}{\omega C}$ (I leads), $X_L = \omega L$ (I lags)

Impedance of Series RLC Circuit: $Z = \sqrt{R^2 + (X_L - X_C)^2}$

Phase in Series RLC Circuit: $\varphi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$

Look at phasor diagram to see this! Pythagorean Theorem