Topics: Ampere's Law
Related Reading:
      Course Notes (Liao et al.):   Sections 9.3 – 9.4; 9.10.2, 9.11.6, 9.11.7
   Serway & Jewett: Sections 30.3 – 30.4

Topic Introduction

In the last class we introduced Ampere's Law.   Today you will get some practice using it, to calculate the magnetic field generated by a cylindrical shell of current and by a slab of current.

Ampere's Law
As previously discussed, Ampere's law rests on the idea that if you have a curl in a magnetic field (that is, if it wraps around in a circle) that the field must be generated by some current source inside that circle (at the center of the curl). So, if we walk around a loop and add up the magnetic field heading in our direction, then if, when we finish walking around, we have seen a net field wrapping in the direction we walked, there must be some current penetrating the loop we just walked around. Mathematically this idea is expressed as:

$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{penetrate}}$, where on the left we are integrating the magnetic field as we walk

around a closed loop, and on the right we add up the total amount of current penetrating the loop.
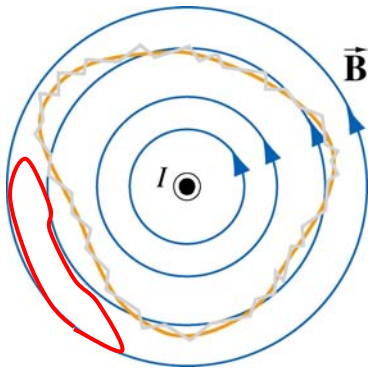
In the example pictured here, a single long wire carries current out of the page. As we discussed in class, this generates a magnetic field looping counter-clockwise around it (blue lines). On the figure we draw two "Amperian Loops." The first loop (yellow) has current I penetrating it. The second loop (red) has no current penetrating it. Note that as you walk around the yellow loop the magnetic field always points in roughly the same direction as the path: $\oint \vec{B} \cdot d\vec{s} \neq 0$, whereas around the red loop sometimes the field points with you, sometimes against you: $\oint \vec{B} \cdot d\vec{s} = 0$.

In Practice
In practice we use Ampere's Law in the same fashion that we used Gauss's Law. There are essentially three symmetric current distributions in which we can use Ampere's Law – for an infinite cylindrical wire (or nested cylindrical shells), for an infinite slab of current (or sets of slabs – like a solenoid), and for a torus (a slinky with the two ends tied together). As with Gauss's law, although the systems can be made more complicated, application of Ampere's Law remains the same:

1) Draw the system so that the current is running perpendicular to the page (into or out of). I **strongly** recommend this step because it means that your Amperian loops will lie in the plane of the page, making them easier to draw. Remember to use circles with dots/x's to indicate currents coming out of/into the page.

2) Determine the symmetry of the system and choose a shape for the Amperian loop(s) – circles for cylinders and toroids, rectangles for slabs. You should determine the direction of the magnetic field everywhere at this point.
3) Determine the regions of space in which the field could be different (e.g. inside and outside of the current)

Then, for each region:
4) Draw the Amperian loop – making sure that on the entire loop (for circular loops) or on each segment of the loop (for rectangular loops) the field is constant and either what you want to know or what you already know (e.g. 0 by symmetry). This is a crucial step since it lets you turn the integral into a simple multiplication.
5) Calculate $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$. If you did step (4) correctly this is just B(Path Length) (summed on each side for rectangular loops). Don't forget that it is a dot product, so that if B is perpendicular to your path the integral is zero.
6) Finally, determine the current punching through your Amperian loop. Often this is just a matter of counting how many wires carrying current I pass through your loop. Sometimes it is slightly more complicated, involving integration of the current density:

$$I = \iint \vec{\mathbf{J}} \cdot d\vec{\mathbf{A}}$$

7) Equate and solve for the magnitude of B. Remember that you got the direction of B in 2.

Important Equations

Ampere's Law: $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{penetrate}$