Problem Solving 4: Calculating Capacitance and Stored Energy

OBJECTIVES

1. To take a sample capacitor and calculate the capacitance of that capacitor.

2. To calculate the energy stored in a capacitor in two ways.

REFERENCE: Section 5.2, 8.02 Course Notes.

PROBLEM SOLVING STRATEGIES (see Section 5.9, 8.02 Course Notes)

(1) Identify the direction of the electric field using symmetry.

(2) Calculate electric field everywhere.

(3) Compute the electric potential difference ΔV.

(4) Calculate the capacitance C using $C = Q / |\Delta V|$.

The Cylindrical Capacitor:
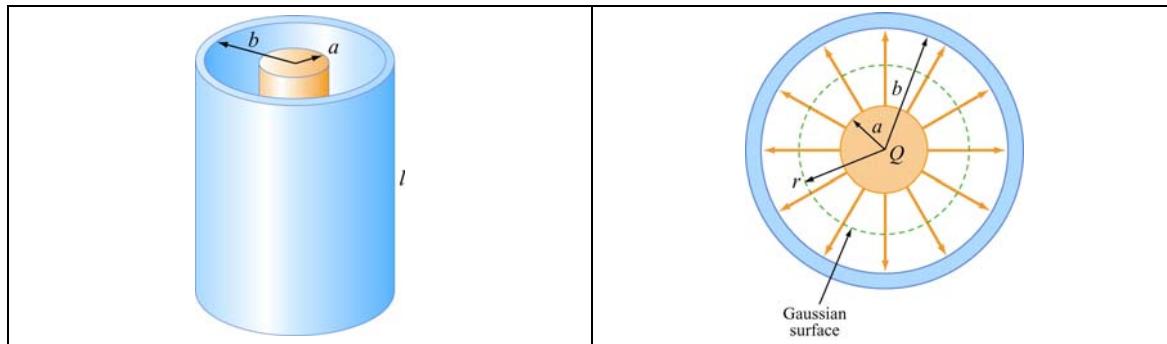
Two concentric spherical cylinders have radii a and b and height l, with $b > a$. The inner cylinder carries total charge Q, and the outer cylinder carries total charge $-Q$. *We ignore end effects*.

Question 1: The Electric Field

Use Gauss's Law to find the direction and magnitude of the electric field in the between the inner and outer cylinders ($a < r < b$). Express your answer in terms of the total charge Q on the inner cylinder cylinder, the radii a and b, the height l, and any other constants which you may find necessary.

Answer (*Write your answer on the tear-sheet at the end!*)

Question 2: Electric Potential Difference (Voltage Difference)

The voltage difference between the cylinders, ΔV, is defined to be the work done per test charge against the electric field in moving a test charge q_t from the inner cylinder to the outer cylinder

$$\Delta V \equiv V(a) - V(b) = -\int_b^a \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} .$$

Find an expression for the voltage difference between the cylinders in terms of the charge Q, the radii a and b, the height l, and any other constants that you may find necessary.

Answer (*Write your answer on the tear-sheet at the end!*)

Question 3: Calculating Capacitance

Our two conducting cylinders form a capacitor. The magnitude of the charge, $|Q|$, on either cylinder is related to the magnitude of the voltage difference between the cylinders according to $|Q| = C|\Delta V|$ where ΔV is the voltage difference across the capacitor and C is the constant of proportionality called the 'capacitance'. The capacitance is determined by the geometrical properties of the two conductors and is therefore independent of the applied voltage difference across the cylinders.

What is the capacitance C of our system of two cylinders? Express your answer in terms, a, and b, the height l, and any other constants which you may find necessary.

Answer (*Write your answer on the tear-sheet at the end!*)

Question 4: Stored Electrostatic Energy

The total electrostatic energy stored in the electric fields is given by the expression,

$$U = \frac{\varepsilon_0}{2} \int_{all\,space} \vec{\mathbf{E}} \cdot \vec{\mathbf{E}} dV_{vol} \ .$$

Starting from your expression for E in question (1), calculate this electrostatic energy and express your answer in terms of Q, a, b, and l (and any other constants which you may find necessary). If you use your expression for C from questions 4 above, can you write you expression in terms of Q and C alone? What is that expression?

Answer (*Write your answer on the tear-sheet at the end!*)

Question 5:   Charging the Capacitor

Suppose instead of using a battery we charge the capacitor ourselves in the following way. We move charge from the inside of the cylinder at $r = b$ to the surface of the cylinder at $r = a$. Suppose we start off with zero charge on the conductors and we move charge for awhile until at time t we have built up a change $q(t)$ on the inner cylinder.

    a. What is the voltage difference between the two cylinders at time t, in terms of C and $q(t)$?

    b. Now we move a very little additional charge dq from the outer to the inner cylinder. How much work dW do we have to do to move that dq from the outer to the inner cylinder, in the presence of the charge $q(t)$ already there, in terms of C, $q(t)$, and dq?

    c. Using your result in (b), calculate the total work we have to do to bring a total charge Q from the outer to the inner cylinder, assuming the cylinders start out uncharged (Hint: integrate with respect to dq from 0 to Q).

    d. Is the work we did in charging the capacitor greater than, equal to, or less than the stored electrostatic energy in the capacitor that you calculated in question 4? Why?

Answer (*Write your answers on the tear-sheet at the end!*)

If You Finish Early, Do The Homework

Problem 1: Capacitors in Series and in Parallel

Consider the circuit shown in the figure, where $C_1 = 6.00\ \mu F$, $C_2 = 3.00\ \mu F$, and $\Delta V = 20.0\,\text{V}$. Capacitor C_1 is first charged by the closing of switch S_1. Switch S_1 is then opened, and the charged capacitor is connected to the uncharged capacitor by the closing of S_2. Calculate the following:

(a) the initial charge acquired by C_1 ,

(b) the final charge on each capacitor.
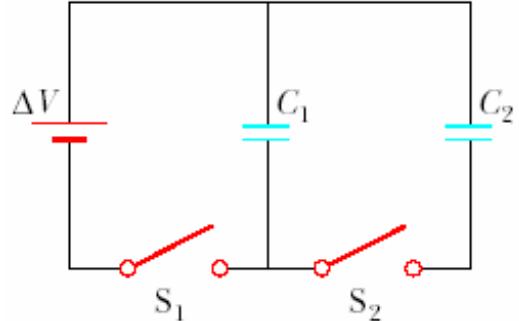
Problem 2: Electrostatic Energy

A cylindrical capacitor consists of a long wire of radius a and length L, with a charge $+Q$ and a concentric cylindrical outer shell of radius $b > a$, length L, with a charge $-Q$.

(a) Find the electric field and energy density at any point in space.

(b) How much energy resides in a cylindrical shell between the conductors of radius r, thickness dr , and volume $2\pi r L\, dr$?

(c) Integrate your expression from part (b) to find the total energy stored in the capacitor and compare your result with that obtained using $U = \dfrac{1}{2}C(\Delta V)^2$.

Problem 3: Capacitors and Dielectrics

(a) Consider a parallel-plate capacitor completely filled with a dielectric material of dielectric constant κ . What is the capacitance of this system?

(b) A parallel-plate capacitor is constructed by filling the space between two square plates with blocks of three dielectric materials, as in the
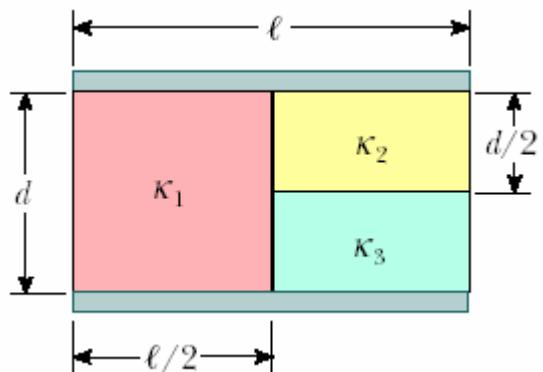
figure below. You may assume that $\ell \gg d$. Find an expression for the capacitance of the device in terms of the plate area A and d, κ_1, κ_2, and κ_3.

Problem 4: Energy stored in Capacitors

A parallel-plate capacitor has fixed charges $+Q$ and $-Q$. The separation of the plates is then doubled.

(a) By what factor does the energy stored in the electric field change?

(b) How much work must be done if the separation of the plates is doubled from d to $2d$? The area of each plate is A.

Consider now a cylindrical capacitor with inner and outer radii a and b, respectively.

(c) Suppose the outer radius b of a cylindrical capacitor is doubled, but the charge is kept constant. By what factor would the stored energy change? Where would the energy come from?

(d) Repeat (c), assuming the voltage remains constant.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Tear off this page and turn it in at the end of class !!!!

Note:
Writing in the name of a student who is not present is a Committee on Discipline offense.

Problem Solving 4: Calculating Capacitance and Stored Energy

Group _____ (e.g. 10A Please Fill Out)

Names _____

_____

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Question 1: The Electric Field

Use Gauss's Law to find the direction and magnitude of the electric field in the between the inner and outer cylinders ($a < r < b$). Express your answer in terms of the total charge Q on the inner cylinder, the radii a and b, the height l, and any other constants which you may find necessary.

Answer:

Question 2: Electric Potential Difference (Voltage Difference):

Find an expression for the voltage difference between the cylinders in terms of the charge Q, a, b, and l (and any other constants which you may find necessary).

Answer:

Question 3: Calculating Capacitance

What is the capacitance C of our system of two cylinders? Express your answer in terms of a, and b, the height l, and any other constants which you may find necessary.

Answer:

Question 4: Stored Electrostatic Energy

What is the total electrostatic energy?   If you use your expression for C from questions 4 above, can you write you expression in terms of Q and C alone? What is that expression?

Answer:

Question 5: Charging the Capacitor

a. What is the voltage difference between the two cylinders at time t, in terms of C and $q(t)$?

b. How much work dW do we have to do to move that dq from the outer to the inner cylinder, in the presence of the charge $q(t)$ already there, in terms of C, $q(t)$, and dq?

c. Using your result in (b), calculate the total work we have to do to bring a total charge Q from the outer to the inner cylinder, assuming the cylinders start out uncharged (Hint: integrate with respect to dq from 0 to Q).

d. Is the work we did in charging the capacitor greater than, equal to, or less than the stored electrostatic energy in the capacitor that you calculated in question 4? Why?