Topics: Maxwell's Equations, EM Radiation & Energy Flow
Related Reading:

Course Notes (Liao et al.):	Chapter 13
Serway & Jewett:	Chapter 34
Giancoli:	Chapter 32

Topic Introduction

Today we will put together much of the physics we have learned in the class to see how electricity and magnetism interact with each other. We begin by finalizing Maxwell's Equations, and then describe their result – electromagnetic (EM) radiation. Finally, we will discuss how energy flows in electric and magnetic fields.

Maxwell's Equations

Now that we have all of Maxwell's equations, let's review:

$$(1)\ \oint\!\!\!\oint_S \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \frac{Q_{in}}{\varepsilon_0} \qquad (2)\ \oint\!\!\!\oint_S \vec{\mathbf{B}}\cdot d\vec{\mathbf{A}} = 0$$

$$(3)\ \oint_C \vec{\mathbf{E}}\cdot d\,\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt} \qquad (4)\ \oint_C \vec{\mathbf{B}}\cdot d\,\vec{\mathbf{s}} = \mu_0 I_{enc} + \mu_0\varepsilon_0\frac{d\Phi_E}{dt}$$

(1) Gauss's Law states that electric charge creates diverging electric fields.
(2) Magnetic Gauss's Law states that there are no magnetic charges (monopoles).
(3) Faraday's Law states that changing magnetic fields can induce electric fields (which curl around the changing flux).
(4) Ampere-Maxwell's Law states that magnetic fields are created both by currents and by changing electric fields, and that in each case the field curls around its creator.

The last piece of this last equation is the one piece you have not seen and we will justify its addition in class. These equations are the cornerstone of the theory of electricity and magnetism. Together with the Lorentz Force $\left(\vec{\mathbf{F}} = q(\vec{\mathbf{E}} + \vec{\mathbf{v}}\times\vec{\mathbf{B}})\right)$ they pretty much describe all of E&M, and from them we can derive mathematically the major equations you learned this semester (like Coulomb's Law and Biot-Savart). People even put them on T-shirts. They are important and you should try hard to keep them in mind.

Electromagnetic Radiation

The fact that changing magnetic fields create electric fields and that changing electric fields create magnetic fields means that oscillating electric and magnetic fields can propagate through space (each pushing forward the other). This is electromagnetic (EM) radiation. It is the single most useful discovery we discuss in this class, not only allowing us to understand natural phenomena, like light, but also to create EM radiation to carry a variety of useful information: radio, broadcast television and cell phone signals, to name a few, are all EM radiation. In order to understand the mathematics of EM radiation you need to understand how to write an equation for a traveling wave (a wave that propagates through space as a function of time). Any function that is written f(x-vt) satisfies this property. As t increases, a function of this form moves to the right (increasing x) with velocity v. You can see this as follows: At t=0 f(0) is at x=0. At a later time t=t, f(0) is at x=vt. That is, the function has moved a distance vt during a time t.

Sinusoidal traveling waves (plane waves) look like waves both as a function of position and as a function of time. If you sit at one position and watch the wave travel by you say that it has a period *T*, inversely related to its frequency *f*, and angular frequency, $\omega\left(T = f^{-1} = 2\pi\omega^{-1}\right)$. If instead you freeze time and look at a wave as a function of position, you say that it has a wavelength λ, inversely related to its wavevector *k* $\left(\lambda = 2\pi k^{-1}\right)$. Using this notation, we can rewrite our function f(x-vt) = $f_0$sin(kx-ωt), where v = ω/k.

We typically treat both electric and magnetic fields as plane waves as they propagate through space (if you have one you must have the other). They travel at the speed of light (v=c). They also obey two more constraints. First, their magnitudes are fixed relative to each other: $E_0 = cB_0$ (check the units!) Secondly, E & B always oscillate at right angles to each other and to their direction of propagation (they are *transverse* waves). That is, if the wave is traveling in the z-direction, and the E field points in the x-direction then the B field must point along the y-direction. More generally we write $\hat{\mathbf{E}} \times \hat{\mathbf{B}} = \hat{\mathbf{p}}$, where $\hat{\mathbf{p}}$ is the direction of propagation.

Energy and the Poynting Vector

As EM Waves travel through space they carry energy with them. This is clearly true – light from the sun warms us up. It also makes sense in light of the fact that energy is stored in electric and magnetic fields, so if those fields move through space then the energy moves with them. It turns out that we can describe how much energy passes through a given area per unit time by the Poynting Vector: $\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}} \times \vec{\mathbf{B}}$. Note that this points in the direction of propagation of the EM waves (from above) which makes sense – the energy is carried in the same direction that the waves are traveling. The Poynting Vector is also useful in thinking about energy in circuit components. For example, consider a cylindrical resistor. The current flows through it in the direction that the electric field is pointing. The B field curls around. The Poynting vector thus points radially *into* the resistor – the resistor consumes energy. We will repeat this exercise for capacitors and inductors in class.

Important Equations

Maxwell's Equations:

(1) $\oiint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{in}}{\varepsilon_0}$ (2) $\oiint_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$

(3) $\oint_C \vec{\mathbf{E}} \cdot d\,\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt}$ (4) $\oint_C \vec{\mathbf{B}} \cdot d\,\vec{\mathbf{s}} = \mu_0 I_{enc} + \mu_0\varepsilon_0\frac{d\Phi_E}{dt}$

EM Plane Waves:

$$\vec{\mathbf{E}}\left(\vec{\mathbf{r}},t\right) = E_0\sin\left(k\hat{\mathbf{p}}\cdot\vec{\mathbf{r}} - \omega t\right)\hat{\mathbf{E}}$$
$$\vec{\mathbf{B}}\left(\vec{\mathbf{r}},t\right) = B_0\sin\left(k\hat{\mathbf{p}}\cdot\vec{\mathbf{r}} - \omega t\right)\hat{\mathbf{B}}$$

with $E_0 = cB_0$; $\hat{\mathbf{E}} \times \hat{\mathbf{B}} = \hat{\mathbf{p}}$; $\omega = ck$

Poynting Vector: $\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}} \times \vec{\mathbf{B}}$