8.02   **Spring 2006**

Experiment 1: Equipotential Lines and Electric Fields

OBJECTIVES

1. To develop an understanding of electric potential and electric fields
2. To better understand the relationship between equipotentials and electric fields
3. To become familiar with the effect of conductors on equipotentials and E fields

PRE-LAB READING
INTRODUCTION

Thus far in class we have talked about fields, both gravitational and electric, and how we can use them to understand how objects can interact at a distance. A charge, for example, creates an electric field around it, which can then exert a force on a second charge which enters that field. In this lab we will study another way of thinking about this interaction through electric potentials.

The Details: Electric Potential (Voltage)

Before discussing electric potential, it is useful to recall the more intuitive concept of potential energy, in particular *gravitational* potential energy. This energy is associated with a mass's position in a gravitational field (its height). The potential energy *difference* between being at two points is defined as the amount of work that must be done to move between them. This then sets the relationship between potential energy and force (and hence field):

$$\Delta U = U_B - U_A = -\int_A^B \vec{\mathbf{F}} \cdot d\,\vec{\mathbf{s}} \quad \Rightarrow \quad \left(\text{in 1D}\right)\ \ F = -\frac{dU}{dz} \qquad (1)$$

We earlier defined fields by breaking a two particle interaction, force, into two single particle interactions, the creation of a field and the "feeling" of that field. In the same way, we can define a potential which is created by a particle (gravitational potential is created by mass, electric potential by charge) and which then gives to other particles a potential energy. So, we define electric potential, *V*, and given the potential can calculate the field:

$$\Delta V = V_B - V_A = -\int_A^B \vec{\mathbf{E}} \cdot d\,\vec{\mathbf{s}} \quad \Rightarrow \quad \left(\text{in 1D}\right)\ \ E = -\frac{dV}{dz}. \qquad (2)$$

Noting the similarity between (1) and (2) and recalling that $F = qE$, the potential energy of a charge in this electric potential must be simply given by $U = qV$.

When thinking about potential it is convenient to think of it as "height" (for gravitational potential in a uniform field, this is nearly precise, since $U = mgh$ and thus the gravitational potential $V = gh$). Electric potential is measured in Volts, and the word "voltage" is often used interchangeably with "potential." You are probably familiar with this terminology from batteries, which maintain fixed potential differences between their two ends (e.g. 9 V in 9 volt batteries, 1.5 V in AAA-D batteries).

Equipotentials and Electric Fields

When trying to picture a potential landscape, a map of equipotential curves – curves along which the potential is equal – can be very helpful. For gravitational potentials these maps are called topographic maps. An example is shown in Fig. 1b.
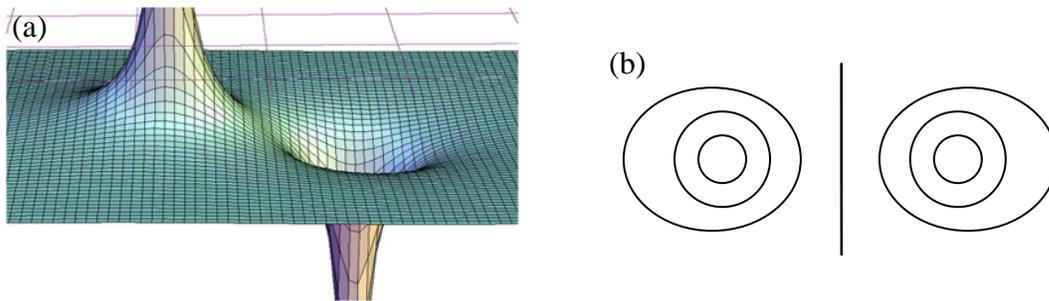
Figure 1: Equipotentials. A potential landscape (pictured in 3D in (a)) can be represented by a series of equipotential lines (b), creating a topographic map of the landscape. The potential ("height") is constant along each of the curves.

Now consider the relationship between equipotentials and fields. At any point in the potential landscape, the field points in the direction that a mass would feel a force if placed there (or that a positive charge would feel a force for electric potentials and fields). So, place a ball at the top of the hill (near the center of the left set of circles in the topographic map of Fig. 1b). Which way does it roll? Downhill! But what direction is that? Perpendicular to the equipotential lines. Why? Equipotential lines are lines of constant height, so moving along them at all does not achieve the objective of going downhill. So the force (and hence field) must point across them, pushing the object downhill. But why exactly perpendicular? Work done on an object changes its potential, so it can take no work to move along an equipotential line. Work is given by the dot product of force and displacement. For this to be zero, the force must be perpendicular to the displacement, that is, force (and hence fields) must be perpendicular to equipotentials.

Note: Potential vs. Potential Difference

Note that in equation (2) we only defined *ΔV*, the potential *difference* between two points, and not the potential *V*. This is because potential is like height – the location we choose to call "zero" is completely arbitrary. In this lab we will choose one location to call zero (the "ground"), and measure potentials relative to the potential at that location.

APPARATUS

1. Conducting Paper Landscapes

To get a better feeling for what equipotential curves look like and how they are related to electric field lines, we will measure sets of equipotential curves for several different potential landscapes. These landscapes are created on special paper (on which you can measure electric potentials) by fixing a potential difference between two conducting shapes on the paper. For reasons that we will discuss later, these conducting shapes are themselves equipotential surfaces, and their shape and relative position determines the electric field and potential everywhere in the landscape. One purpose of this lab is to develop an intuition for how this works. There are four landscapes to choose from (Fig. 2), and you will measure equipotentials on two of them (one from Fig. 1a, b and one from Fig. 1c, d).
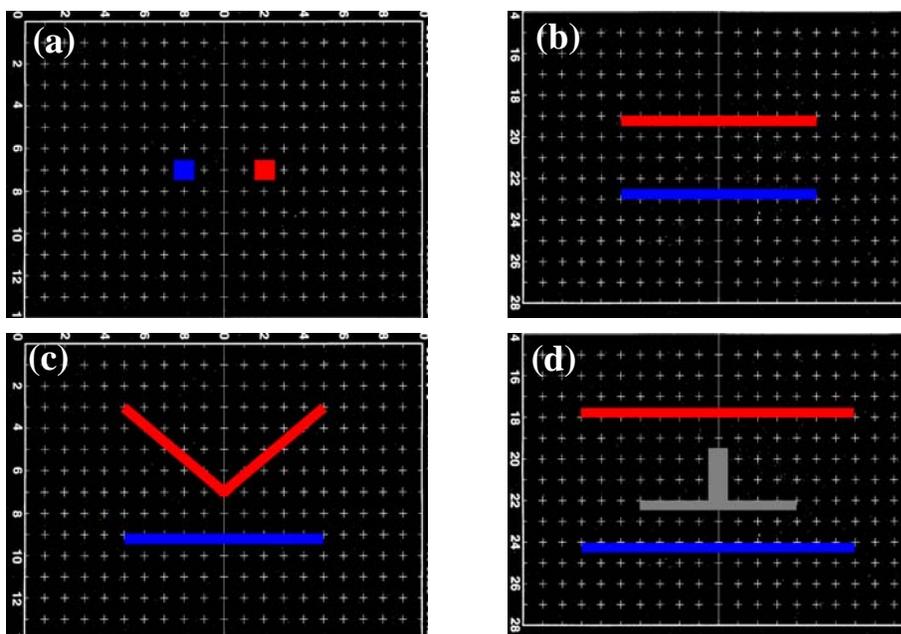
Figure 2 Conducting Paper Landscapes. Each of the four landscapes – the "standard" (a) dipole and (b) parallel plates, and the "non-standard" (c) bent plate and (d) filled plates – consists of two conductors which will be connected to the positive (red) and ground (blue) terminals of a battery. In (d) there is an additional conductor which is free to float to whatever potential is required. The pads are painted on conducting paper with a 1 cm grid.

2. Science Workshop 750 Interface

In this lab we will use the Science Workshop 750 interface (Fig. 3) both to create the potential landscapes (using the "OUPUT" connections that act like a battery) and to measure the potential at various locations in that landscape using a voltage sensor (see below). There are two connections to the output, just like there are two sides of a battery.

The one marked with a sin wave is the positive terminal and the one marked with a triangle (the "ground symbol") is the negative terminal. The potential of this terminal is what we will call zero. We will set the potential difference between the two to 5 V.

Figure 3 The *Science Workshop* 750 Interface.

3. Voltage Sensor

In order to measure the potential as a function of position we use a voltage sensor (Fig. 4). One side of the sensor plugs into Channel A on the 750, the other has two leads, red and black. When the 750 records the "potential," it really measures the potential difference between the two leads, the potential at the red lead minus that at the black lead.

Figure 4 Voltage Sensor

GENERALIZED PROCEDURE

For each of the two landscapes that you choose, you will find at least four equipotential contours by searching for points in the landscape at the same potential using the voltage sensor. After recording these curves, you will draw several electric field lines, making use of the fact that they are everywhere perpendicular to equipotential contours.

END OF PRE-LAB READING

Expt. 1: Equipotential Lines and Electric Fields Pre-Lab Questions

Answer these questions on a separate sheet of paper and turn them in before the lab

1. Equipotentials Curves – Reading Topographic Maps

Below is a topographic map of a 0.4 mi square region of San Francisco. The contours shown are separated by heights of 25 feet (so from 375 feet to 175 feet above sea level for the region shown)
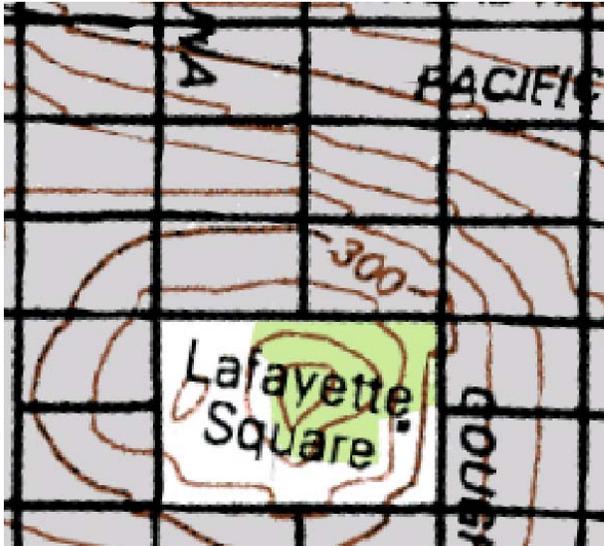

From left to right, the NS streets shown are Buchanan, Laguna, Octavia, Gough and Franklin. From top to bottom, the EW streets shown are Broadway, Pacific, Jackson, Washington, Clay (which stops on either side of the park) and Sacramento.

(a) In the part of town shown in the above map, which street(s) have the steepest runs? Which have the most level sections? How do you know?

(b) How steep is the steepest street at its steepest (what is its slope in ft/mi)?

(c) Which would take more work (in the physics sense): walking 3 blocks south from Laguna and Jackson or 1 block west from Clay and Franklin?

2. Equipotentials, Electric Fields and Charge
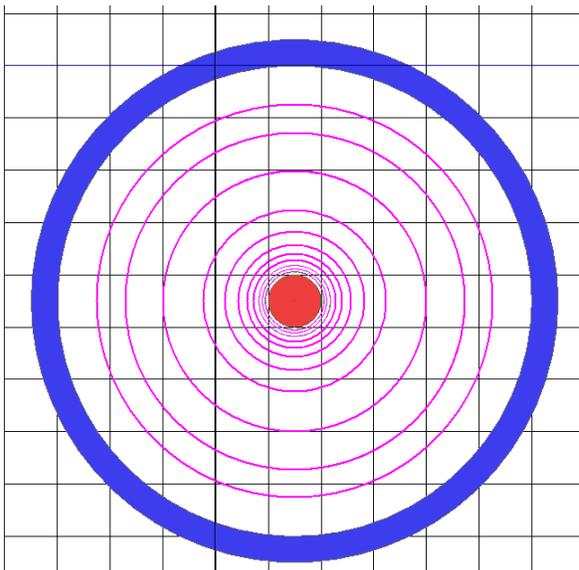
One group did this lab and measured the equipotentials for a slightly different potential landscape then the ones you have been given (although still on a 1 cm grid) and using +10 V rather than +5V.

Note that they went a little overboard and marked equipotential curves (the magenta circles) at V = 0.25 V, 0.5V and then from V = 1 V to V = 10 V in 1 V increments.

They followed the convention that red was their positive electrode ($V = +10$ V) and blue was ground ($V = 0$ V).

(a) Copy the above figure and sketch eight electric field lines on it (equally spaced around the inner conductor).

(b) What, approximately, is the magnitude of the electric field at $r = 1$ cm, 2 cm, and 3 cm, where r is measured from the center of the inner conductor? You should express the field in V/cm. (HINT: The field is the local slope (derivative) of the potential. Also, if you choose to use a ruler realize that the above reproduction of this group's results is not the same size as the original, where the grid size was 1 cm).

(c) What is the relationship between the density of the equipotential lines, the density of the electric field lines, and the strength of the electric field?

(d) Plot the field strength vs. $1/r^2$ for the three points from part (b). If the field were created by a single point charge what shape should this sketch be? Is it?

(e) Approximately how much charge was on the inner conductor when the group made their measurements? Make sure you include the sign as well as the amplitude. (HINT: Use the sketch of (d))

IN-LAB ACTIVITIES

EXPERIMENTAL SETUP

1. Download the LabView file from the web and save the file to your desktop (right click on the link and choose "Save Target As"). Start LabView by double clicking

2. Connect cables to the output of the 750 (red to the sin wave marked output, black to ground). One member of the group will hold these wires to the two conductors while another maps out the equipotentials.

3. Connect the Voltage Sensor to Analog Channel A on the 750 Interface

4. Connect the black lead of the voltage sensor to the black output (the ground). You will use the red lead to measure the potential around your landscapes.

MEASUREMENTS

Part 1: "Standard" Configuration

1. Choose one of the two "standard" conducting paper landscapes (the dipole or parallel plate configuration)

2. Use the voltage connectors to make contacts to the two conducting pads

3. Press the green "Go" button above the graph to energize the battery and begin recording the potential of the red lead (relative to the black lead = ground).

4. Measure the potential of both conducting pads to confirm that they are properly connected (one should be at +5 V, the other at 0 V), and that they are indeed equipotential objects (we will explain why next week).

5. Now, try to find some location on the paper that is at about +1 V (don't worry about being too precise). Mark this point on the plot on the next page.
   ## Do NOT write on the conducting paper

6. Find another 1 V point, about 1 cm away. Continue until you have closed the curve or left the page. Sketch and label this equipotential curve.

7. Repeat this process to find equipotentials at 2 V, 3 V, and 4 V. Work pretty fast; it's more important to think about what these lines mean than it is to draw them perfectly. Think about what you are doing – are there symmetries that you can exploit to make this task easier?
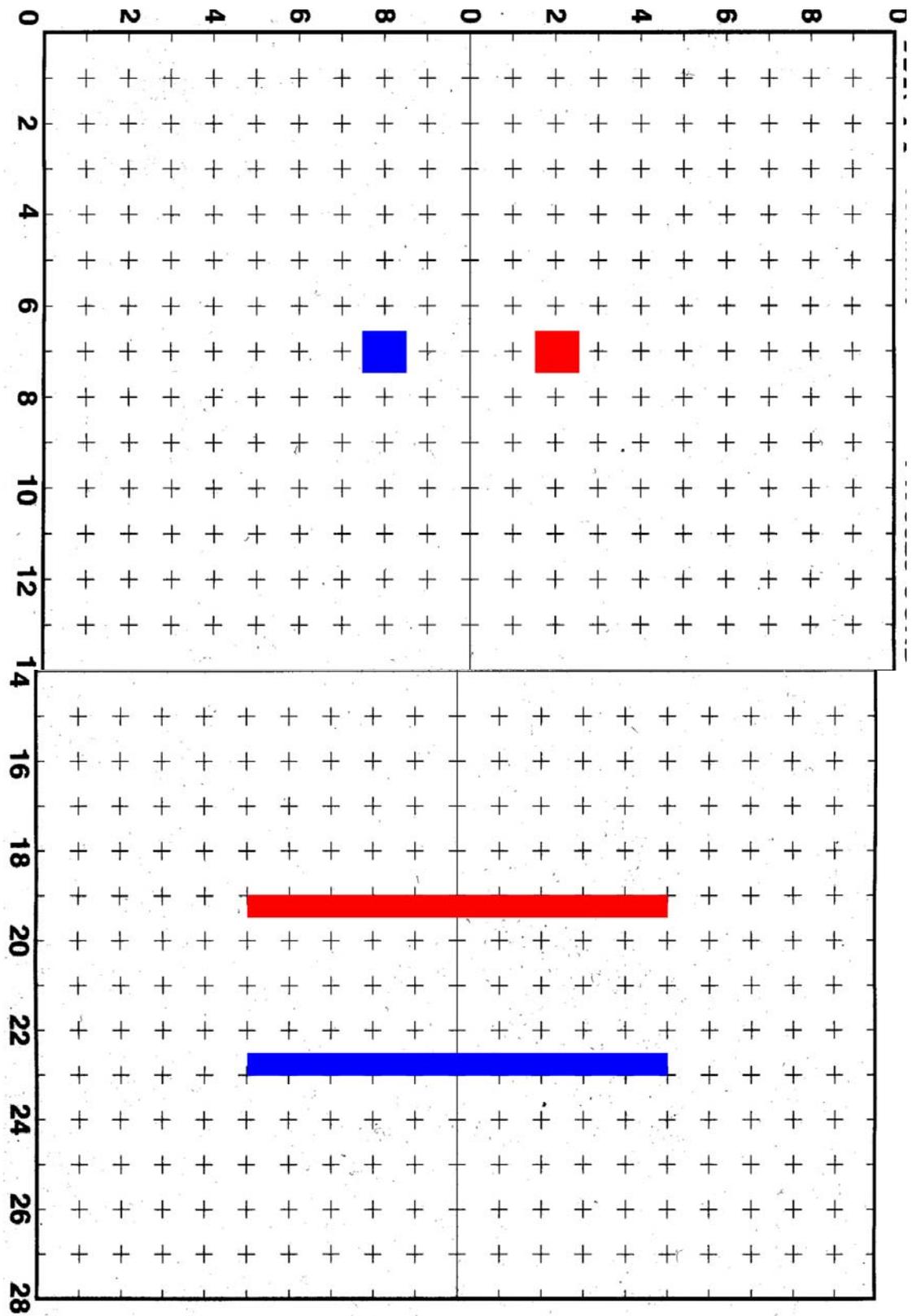
Question 1:

Sketch in a set of electric field lines (~ ten) on your plot of equipotentials on the previous page. Where do the field lines begin and end? If they are equally spaced at their beginning, are they equally spaced at the end? Along the way? Why?

Question 2:

What, approximately, is the potential midway between the two conductors?
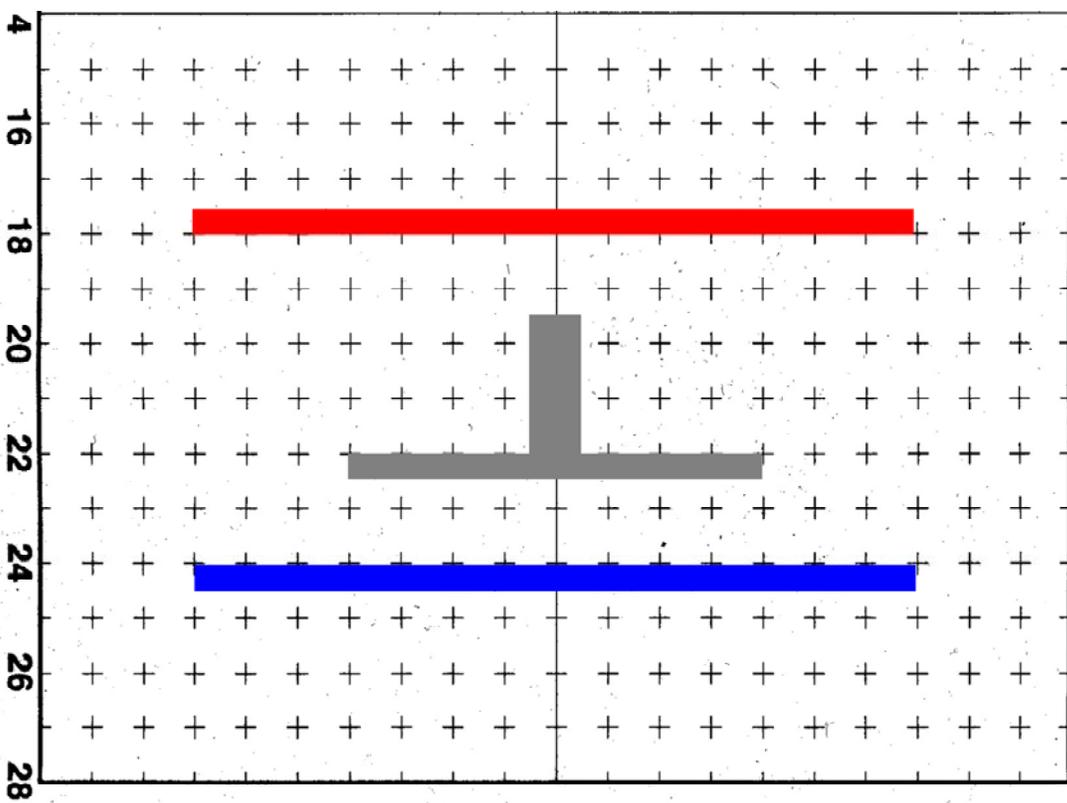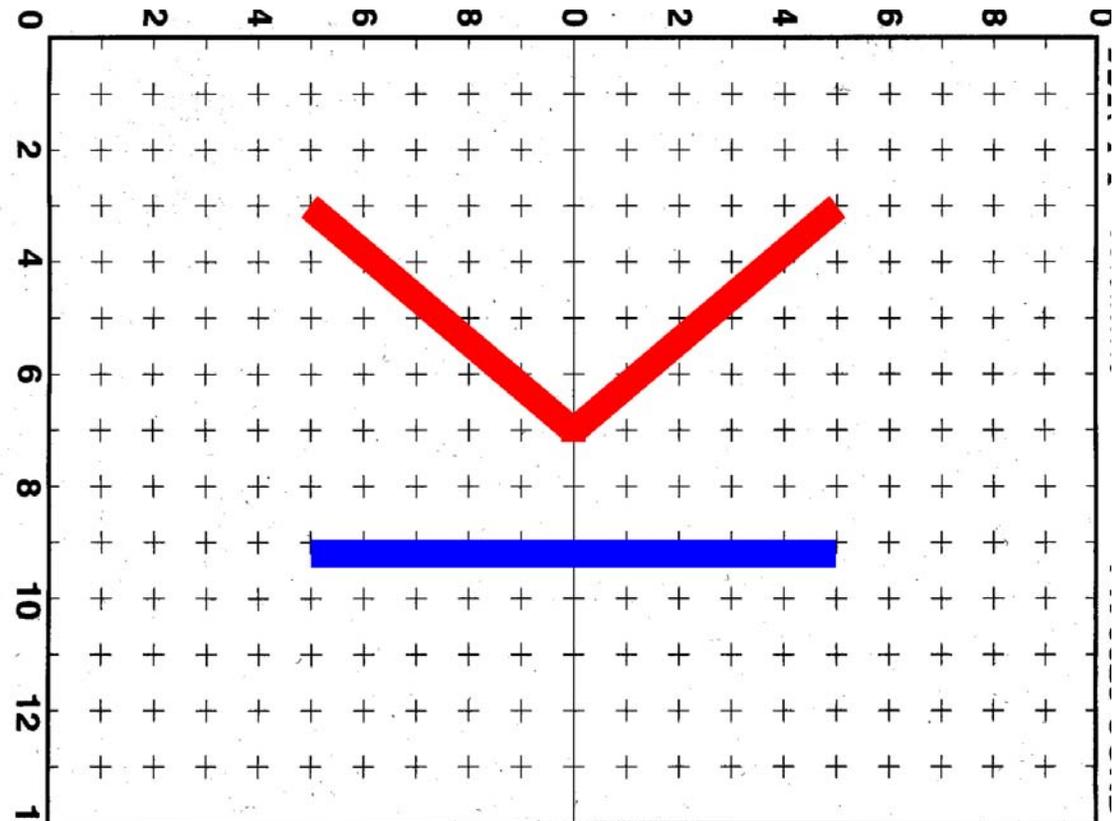
Question 3:

What, approximately, is the strength of the electric field midway between the two conductors? You may find it easier to answer this question if you just measure the potential at a few points near the center.

Part 2: "Non-Standard" Configuration

1. Choose one of the two "non-standard" conducting paper landscapes (the bent plate or filled plates configuration)

2. Use the voltage connectors to make contacts to the two conducting pads (for the filled plates, the center pad does *not* have a connection to it)

3. Press the green "Go" button above the graph to energize the battery and begin recording the potential of the red lead (relative to the black lead = ground).

4. Confirm that everything is properly connected by measuring the potential on the two connected pads, then record a set of equipotential curves following the same procedure of part 1.

"Non-Standard" Configurations

Question 4:

Sketch in a set of electric field lines on your plot of equipotentials on the previous page. Where is the electric field the strongest? What, approximately, is its magnitude?

Question 5:

Where is the electric field the most uniform? How can you tell?

Further Questions (for experimentation, thought, future exam questions…)

- What changes if you switch which conducting pad is at +5 V and which is ground?
- What if you forget to connect the ground lead?
- If you rest your hand on the paper while making measurements, does it affect the readings? Why or why not?
- If you wanted to push a charge along one of the field lines from one conductor to the other, how does the choice of field line affect the amount of work required?
- The potential is everywhere the same on an equipotential line. Is the electric field everywhere the same on an electric field line?