Class 15: Outline

Hour 1:

Magnetic Force

Expt. 6: Magnetic Force

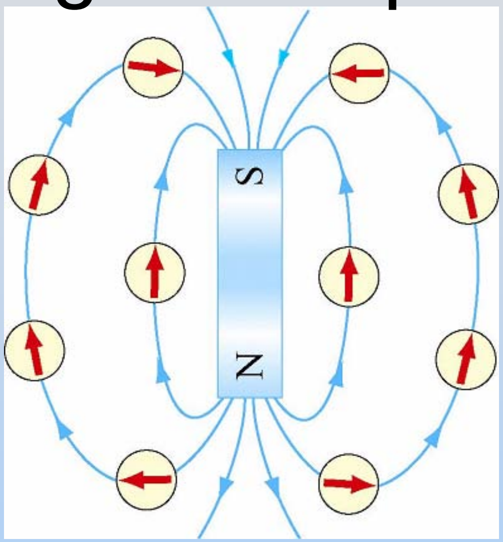
Hour 2:

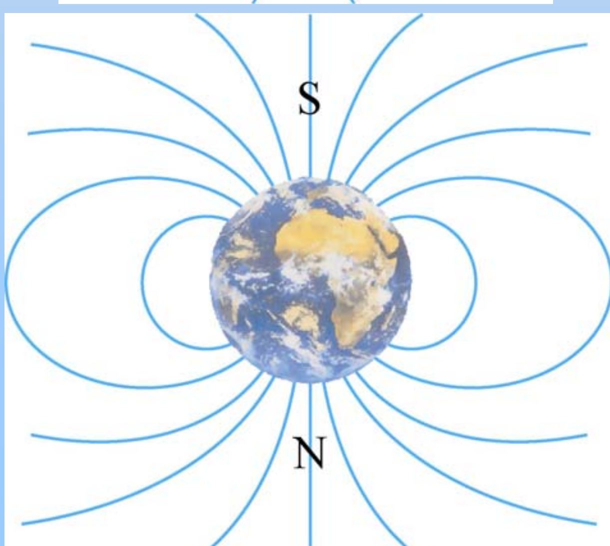
Creating B Fields: Biot-Savart

Last Time:
Magnetic Fields
&
Magnetic Dipoles

Magnetic Fields

Magnetic Dipoles Create and Feel B Fields:





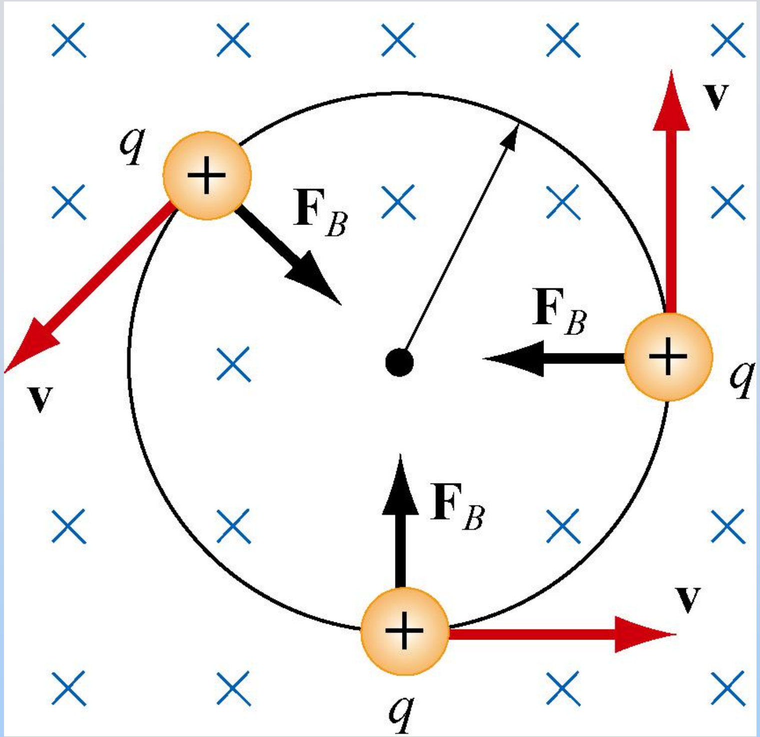
Also saw that

moving charges feel a force:

$$\vec{\mathbf{F}}_B = q\,\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

What Kind of Motion Does this Lead to?

Cyclotron Motion



(1) r : radius of the circle

$$qvB = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{qB}$$

(2) T : period of the motion

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$$

(3) ω : cyclotron frequency

$$\boxed{\omega = 2\pi f = \frac{v}{r} = \frac{qB}{m}}$$

Current Carrying Wires

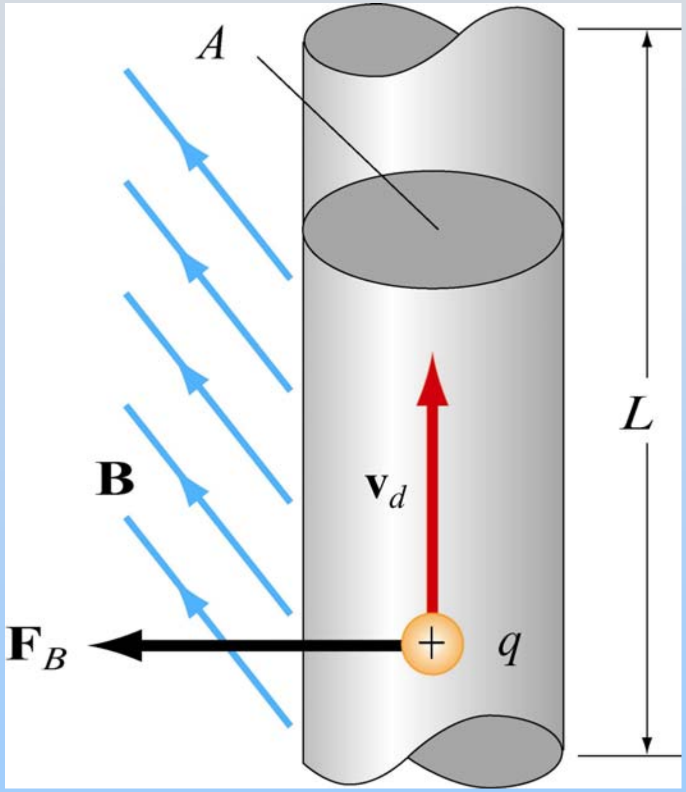
Magnetic Force on Current-Carrying Wire

$$\vec{\mathbf{F}}_B = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

$$= \left(\text{charge}\right)\frac{\text{m}}{\text{s}} \times \vec{\mathbf{B}}$$

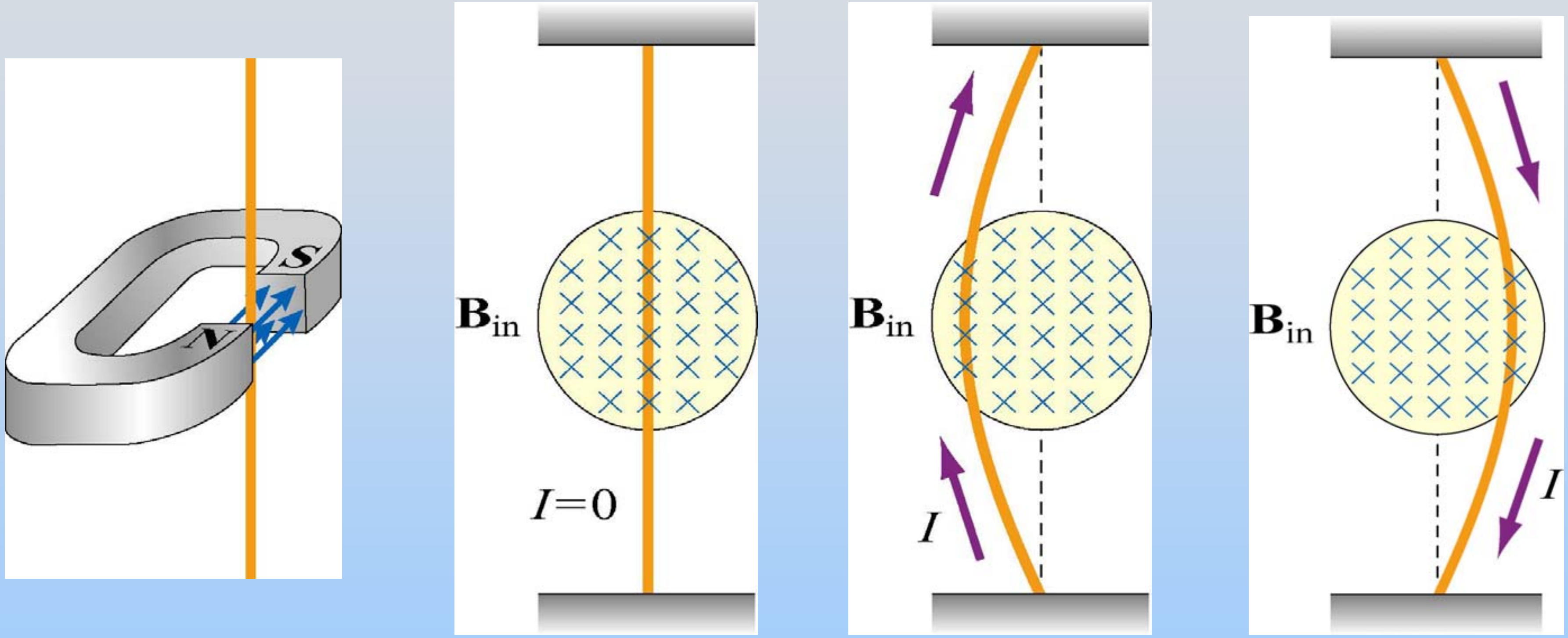
$$= \frac{\text{charge}}{\text{s}}\,\text{m} \times \vec{\mathbf{B}}$$

$$\boxed{\vec{\mathbf{F}}_B = I\left(\vec{\mathbf{L}} \times \vec{\mathbf{B}}\right)}$$



Demonstration:
Jumping Wire

Magnetic Force on Current-Carrying Wire



Current is moving charges, and we know that moving charges **feel** a force in a magnetic field

PRS Questions:
5 Predictions For Experiment 6

Experiment 6:
Magnetic Force

Mid-term Course Evaluation

Lab Summary: Currents FEEL Forces in Magnetic Fields

Question: What happens if currents are next to each other?

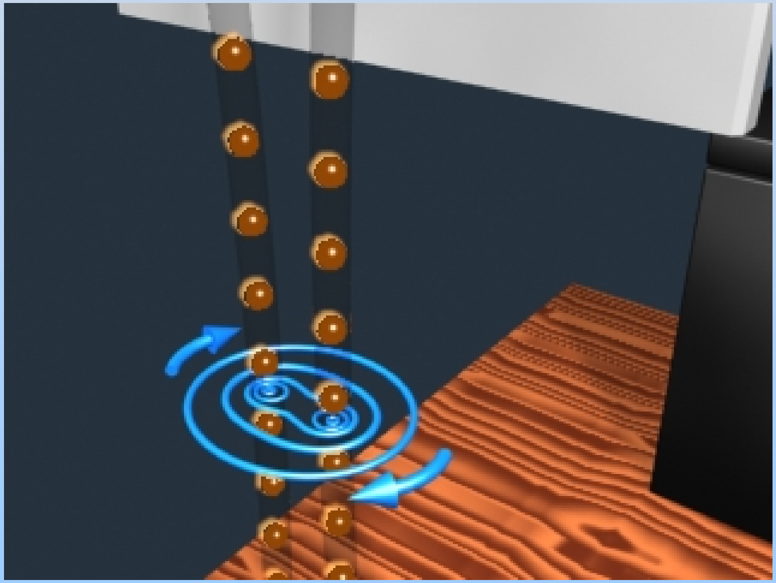
Demonstration:
Parallel & Anti-Parallel Currents

How Do They Interact?

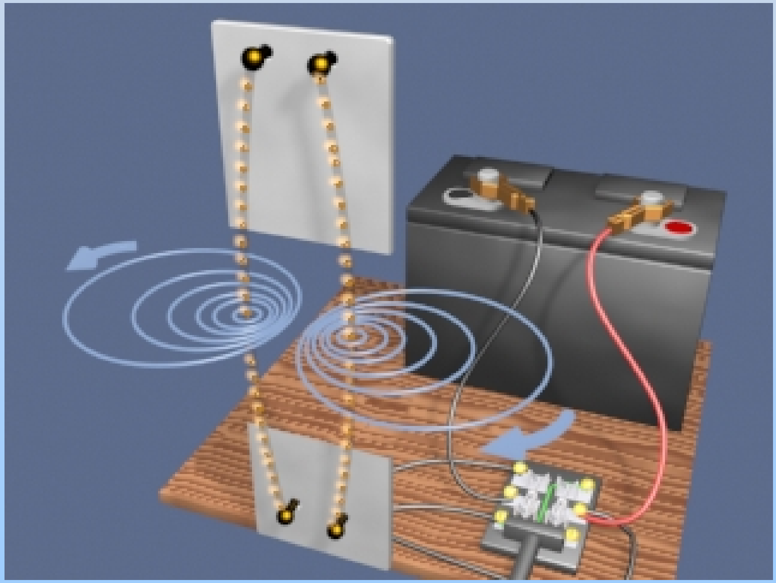
Moving charges also **create** magnetic fields!

The current in one wire *creates* a magnetic field that is *felt* by the other wire.

This is the rest of today's focus



(http://ocw.mit.edu/ans7870/8/8.02T/f04/visualizations/magnetostatics/13-ParallelWires/13-Parallel_Wires_320_f185.html)

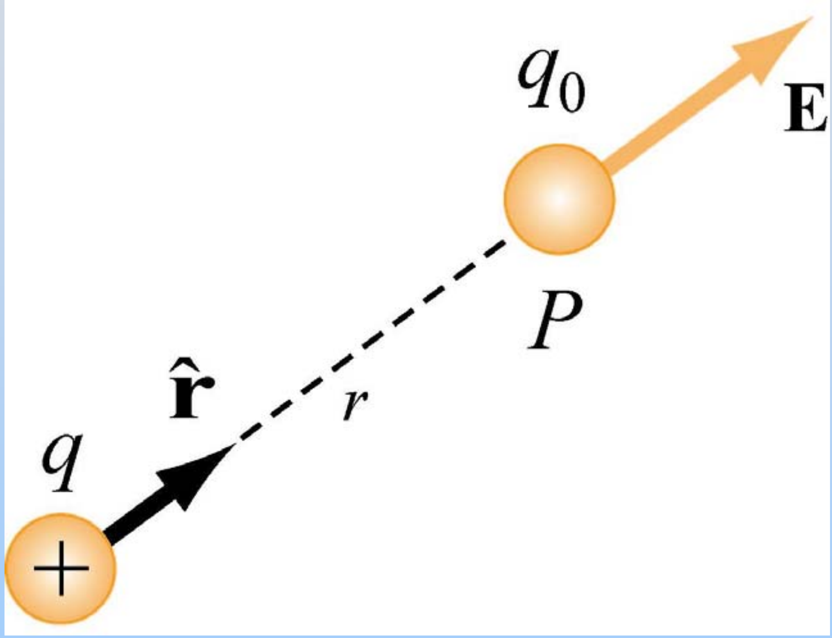


(http://ocw.mit.edu/ans7870/8/8.02T/f04/visualizations/magnetostatics/14-SeriesWires/14-Series_320.html)

Sources of Magnetic Fields: Biot-Savart

Electric Field Of Point Charge

An electric charge produces an electric field:

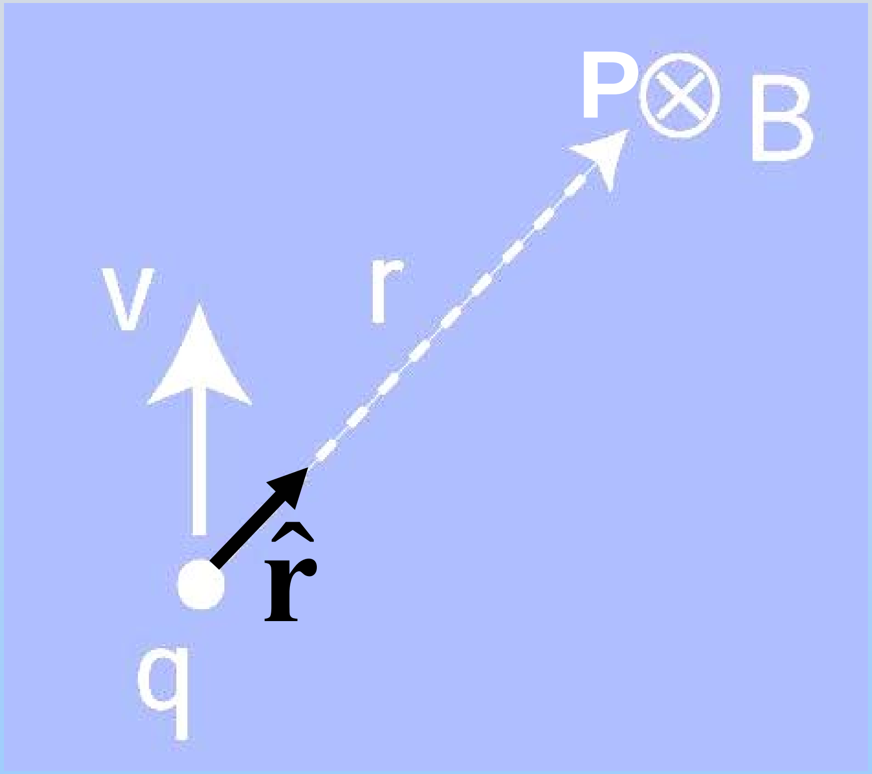


$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}\hat{\mathbf{r}}$$

$\hat{\mathbf{r}}$: unit vector directed from q to *P*

Magnetic Field Of Moving Charge

Moving charge with velocity v produces magnetic field:



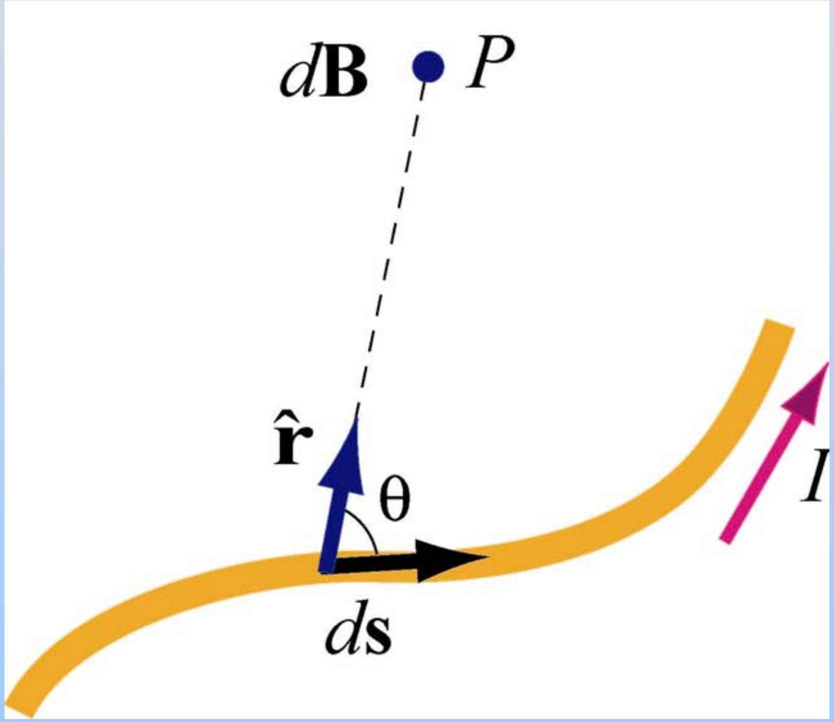
$$\vec{\mathbf{B}} = \frac{\mu_o}{4\pi} \frac{q\,\vec{\mathbf{v}} \times \hat{\mathbf{r}}}{r^2}$$

$\hat{\mathbf{r}}$: unit vector directed from q to P

$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$ permeability of free space

The Biot-Savart Law

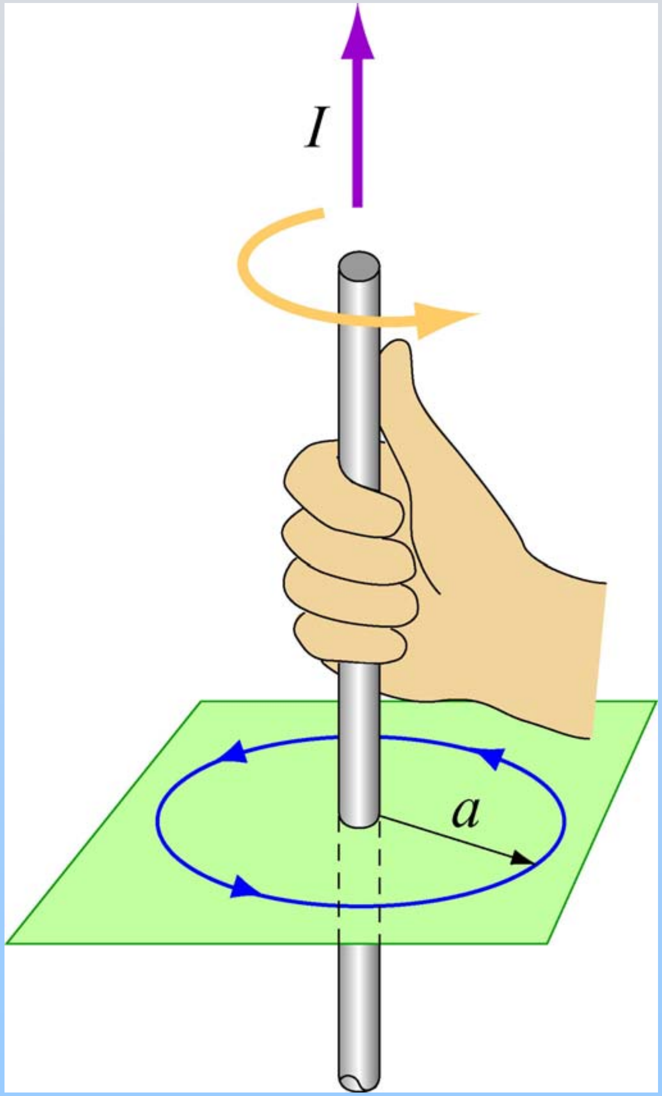
Current element of length ds carrying current I produces a magnetic field:

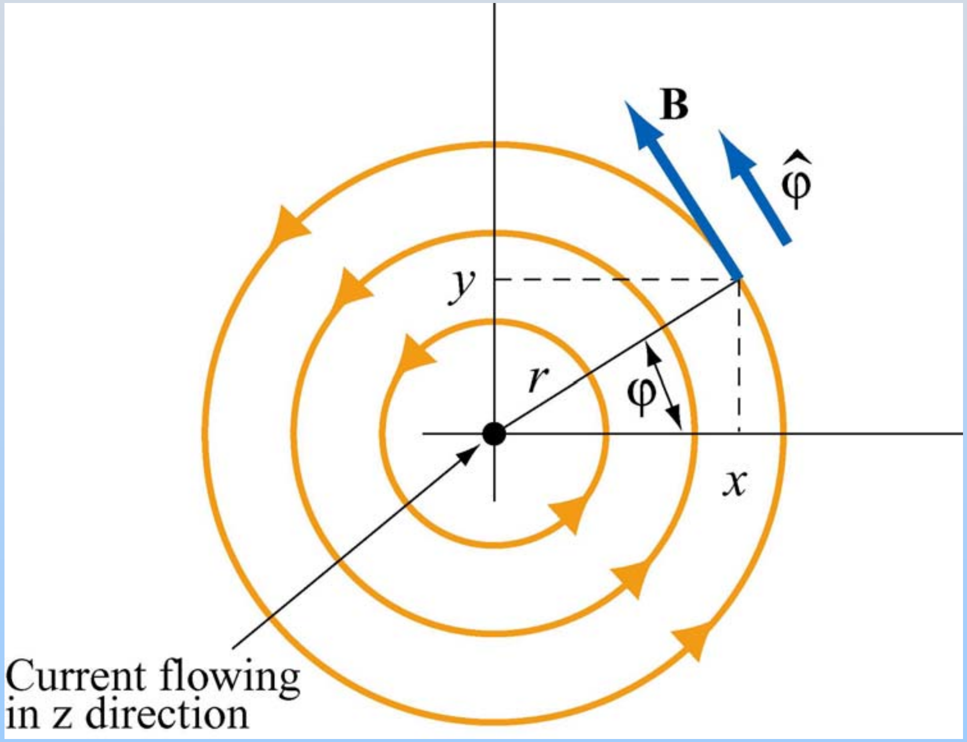


$$\mathbf{d}\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{\mathbf{s}}\times\hat{\mathbf{r}}}{r^2}$$

(http://ocw.mit.edu/ans7870/8/8.02T/f04/visualizations/magnetostatics/03-CurrentElement3d/03-cElement320.html)

The Right-Hand Rule #2

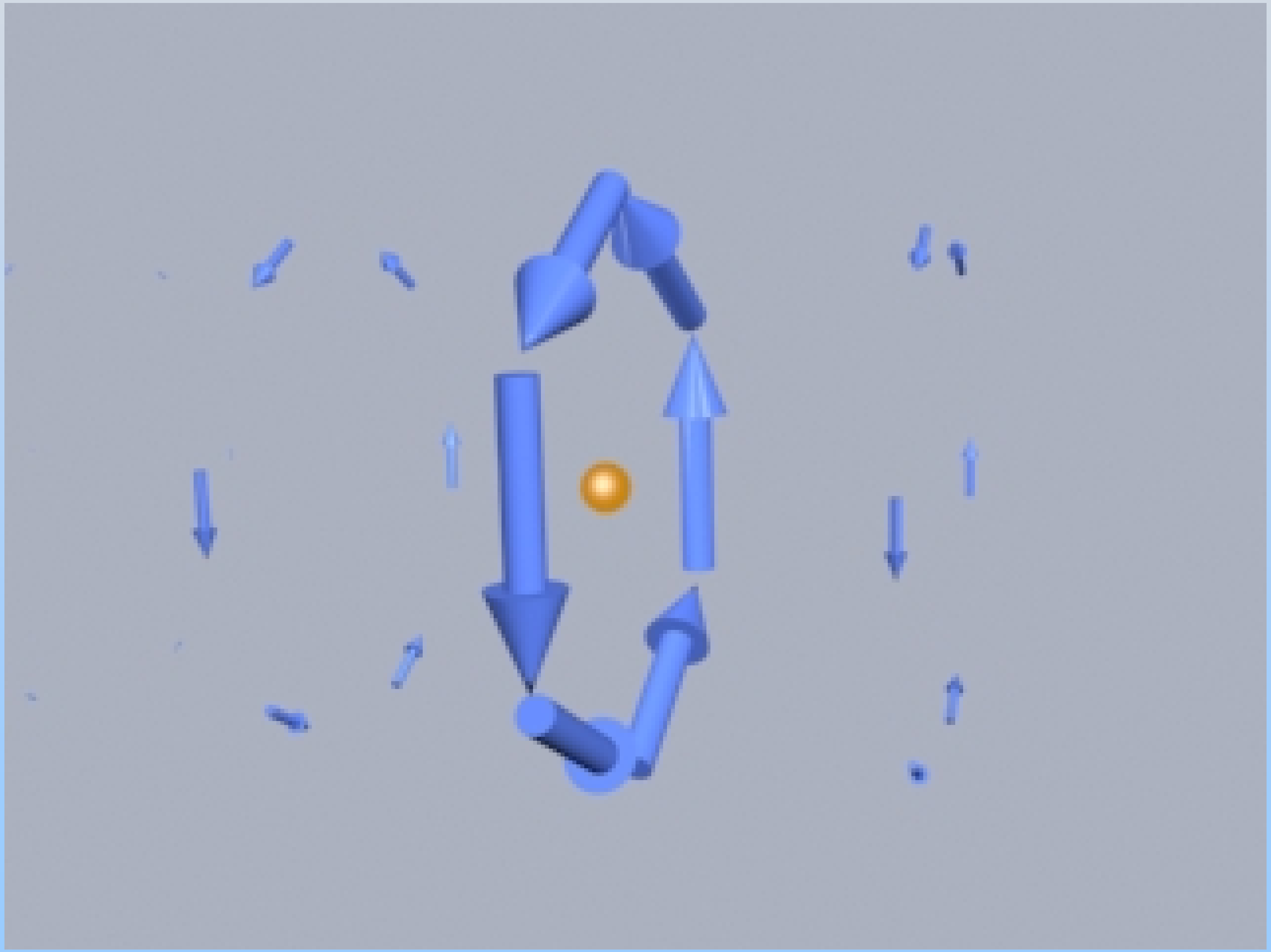




$$\hat{\mathbf{z}} \times \hat{\boldsymbol{\rho}} = \hat{\boldsymbol{\varphi}}$$

Animation: Field Generated by a Moving Charge

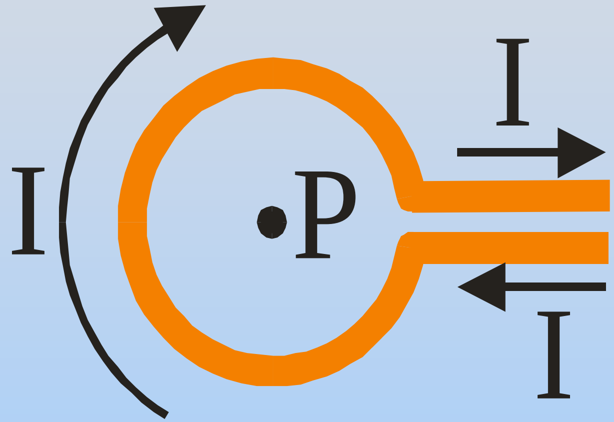
(http://ocw.mit.edu/ans7870/8/8.02T/f04/visualizations/magnetostatics/01-MovingChargePosMag/01-MovChrgMagPos_f223_320.html)



Demonstration:
Field Generated by Wire

Example : Coil of Radius *R*

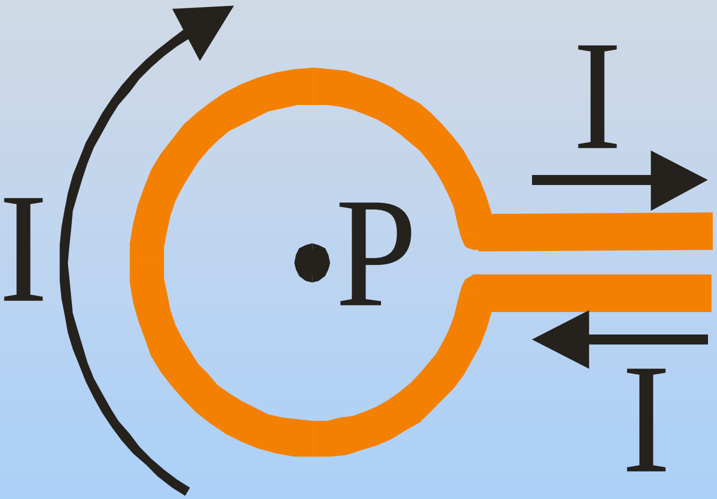
Consider a coil with radius *R* and current *I*



Find the magnetic field B at the center (P)

Example : Coil of Radius *R*

Consider a coil with radius *R* and current *I*

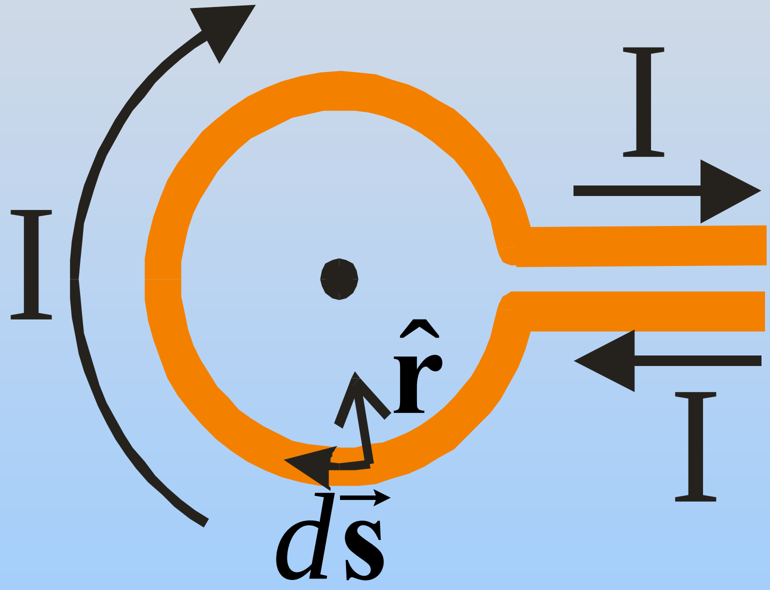


1) Think about it:
   - Legs contribute nothing
     I parallel to *r*
   - Ring makes field into page
2) Choose a ds
3) Pick your coordinates
4) Write Biot-Savart

Example : Coil of Radius *R*

In the circular part of the coil…

$$d\vec{\mathbf{s}} \perp \hat{\mathbf{r}} \quad \rightarrow \quad |d\vec{\mathbf{s}} \times \hat{\mathbf{r}}| = ds$$



Biot-Savart:

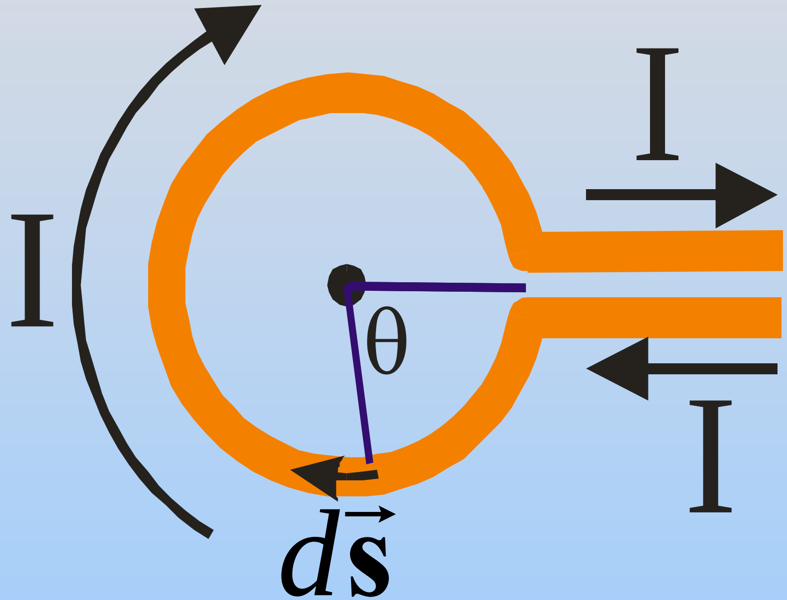
$$dB = \frac{\mu_0 I}{4\pi} \frac{\left| d\vec{\mathbf{s}} \times \hat{\mathbf{r}} \right|}{r^2} = \frac{\mu_0 I}{4\pi} \frac{ds}{r^2}$$

$$= \frac{\mu_0 I}{4\pi} \frac{R\, d\theta}{R^2}$$

$$= \frac{\mu_0 I}{4\pi} \frac{d\theta}{R}$$

Example : Coil of Radius *R*

Consider a coil with radius *R* and current *I*



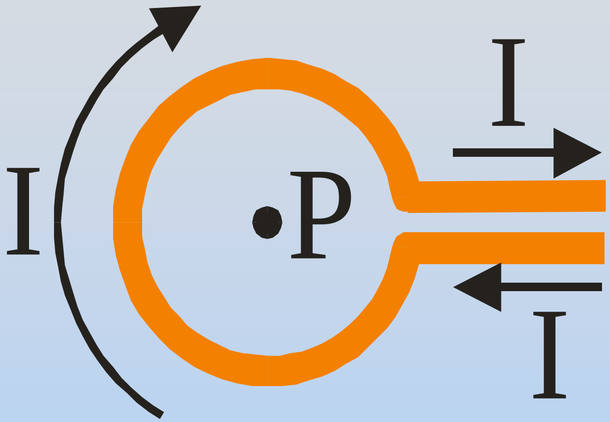
$$dB = \frac{\mu_0 I}{4\pi}\frac{d\theta}{R}$$

$$B = \int dB = \int_0^{2\pi} \frac{\mu_0 I}{4\pi}\frac{d\theta}{R}$$

$$= \frac{\mu_0 I}{4\pi R}\int_0^{2\pi} d\theta = \frac{\mu_0 I}{4\pi R}(2\pi)$$

$$\vec{\mathbf{B}} = \frac{\mu_0 I}{2R} \text{into page}$$

Example : Coil of Radius *R*



$$\vec{\mathbf{B}} = \frac{\mu_0 I}{2R} \text{ into page}$$

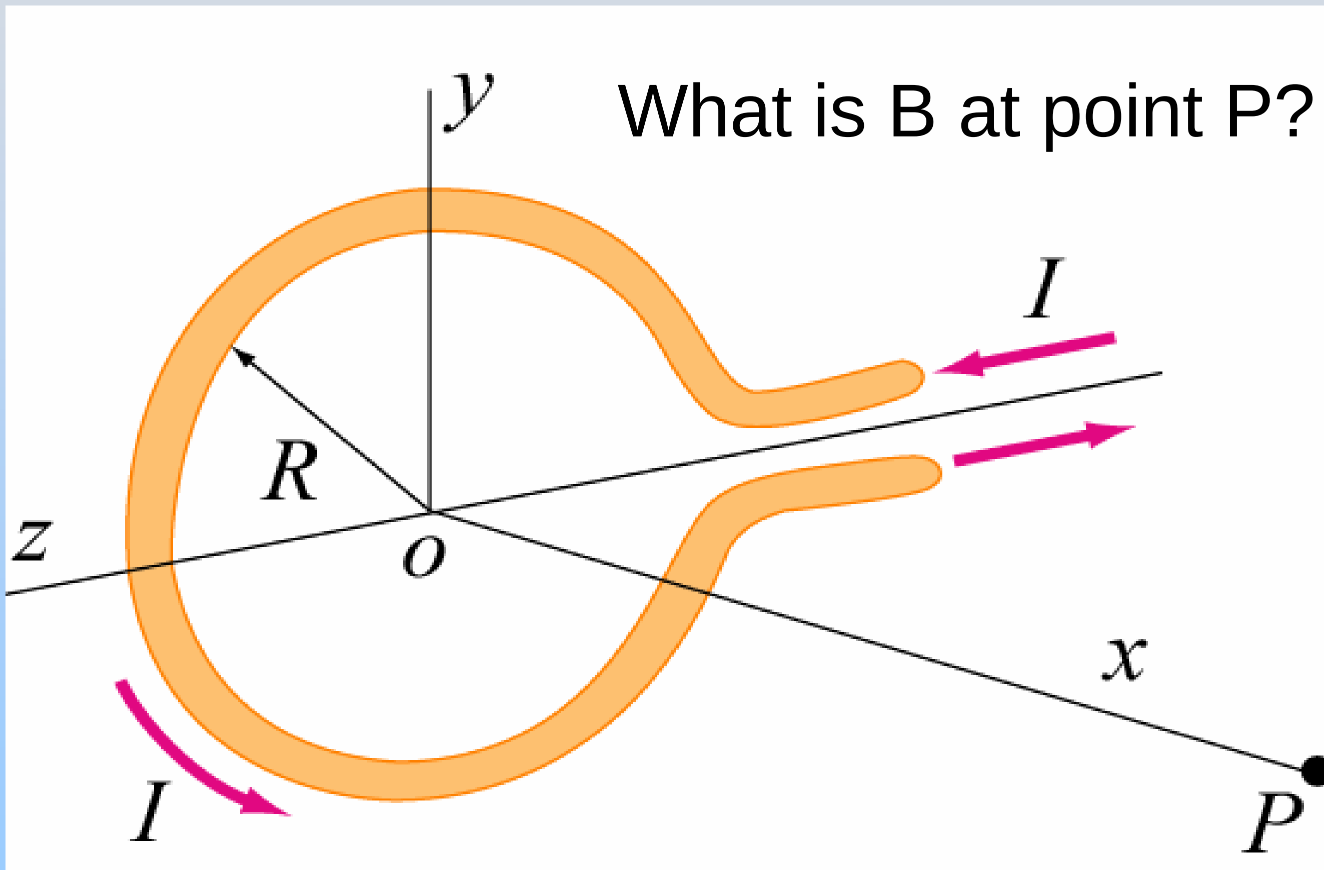
Notes:

- This is an EASY Biot-Savart problem:
  - No vectors involved
- This is what I would expect on exam

PRS Questions:
B fields Generated by Currents

Group Problem:
B Field from Coil of Radius R

Consider a coil with radius *R* and carrying a current *I*



WARNING:

This is much harder than what I just did! Why??

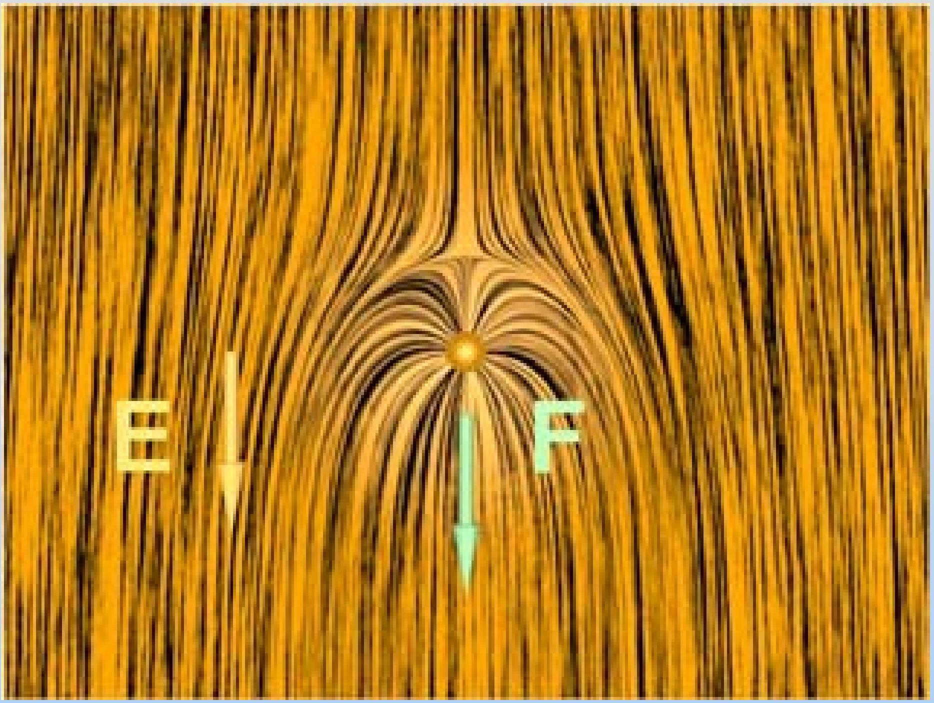
Field Pressures and Tensions: A Way To Understand the qVxB Magnetic Force

Tension and Pressures Transmitted by E and B

Fields **(E or B):**

- Transmit tension along field direction (Field lines want to pull straight)
- Exert pressure perpendicular to field (Field lines repel)

Example of E Pressure/Tension



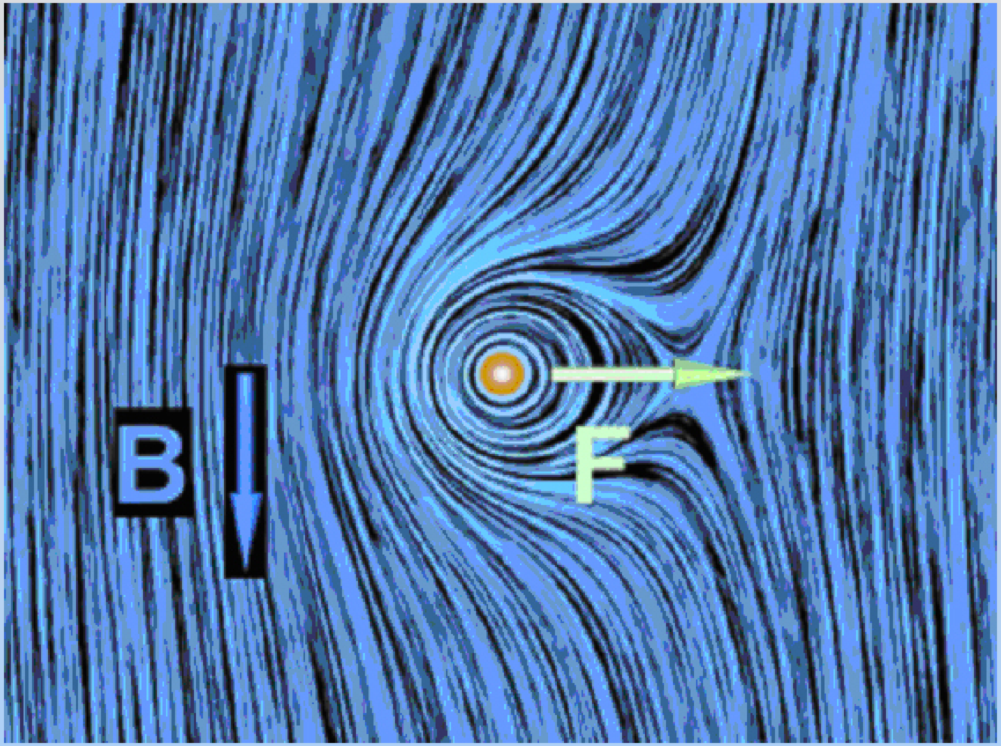
(http://ocw.mit.edu/ans7870/8/8.02T/f04/visualizations/electrostatics/11-forceq/11-ForceQ_f0_320.html)

Positive charge in uniform (downward) E field

Electric force on the charge is combination of

1. Pressure pushing down from top
2. Tension pulling down towards bottom

Example of B Pressure/Tension



(http://ocw.mit.edu/ans7870/8/8.02T/f04/visualizations/magnetostatics/10-forcemovingq/10-ForceMovingQ_f0_320.html)

Positive charge moving out of page in uniform (downwards) B field. Magnetic force combines:

1. Pressure pushing from left
2. Tension pulling to right