

When solving physics problems, we have fundamental principles.

For instance, we have Newton's second law.

And we also have an energy principle, work non-conservative equals the change in kinetic energy plus the change in potential energy.

And many times students are challenged by-- do I use this law?

Do I use that law?

Why do I use one or the other?

Do I use both at the same time?

Can I get the same answers, et cetera.

Now I want to illustrate a particular category of problems where we have some type of circular motion, and why we need to use both of these laws.

So let's consider, let's go back to our dome example at MIT.

And let's say we have an object that's on the dome.

And now let's write out a free body diagram for this object.

So we have our gravitational force.

We have a normal force.

And here, let's just assume that our dome is frictionless, for simplicity.

So what you see here is that the object is displacing in a direction-- now here if I choose \hat{r} and $\hat{\theta}$, and if I choose a coordinate system where I take my positive angle θ this way, that the normal force and the displacement are perpendicular.

Now what that means is that the work done by the normal force, $F_{\text{normal}} \cdot ds$ is 0, because well, we're calling F_{normal} equal to the normal force and that the normal force is perpendicular to displacement.

So what that means is all information about the normal force is not included in the energy principle.

In fact, the energy principle is-- the work that's done is just the amount-- this is angle theta, so this is angle theta-- so the actual work done by the gravitational force g is the component of g $mg \sin \theta$ times the displacement ds .

So the only work that's appearing here, and this is conservative.

And so when we integrated this work and got that the work done by the gravitational force is minus the change in potential energy, that's showing up here.

So the part of the force that's in the direction of the motion is giving us the energy condition.

But we're losing all information about the forces in the radial direction.

So because also what we need is Newton's second law in the direction perpendicular to the displacement.

Displacement is in the $\hat{\theta}$ direction, so we need Newton's second law in the radial direction.

And that we have r hat, is we have a normal force minus $mg \cos \theta$.

And the object is undergoing circular motion, so it's equal to minus mv^2 over R , where R is the radius of the dome.

This equation is not at all included in the energy condition.

You may say, well what about the tangential Newton's second law, which is $mg \sin \theta = mR \frac{d^2 \theta}{dt^2}$.

But it's precisely this equation that's integrated with respect to the displacement and that gives us our energy principle.

So in summary, the energy principle is the integration of Newton's second law in the direction of motion.

And we're completely missing the application of Newton's second law in the direction perpendicular to the motion.

Energy does not account for that.

And that's why we needed separately to apply both of the principles of energy and Newton's second law in the radial direction in order to figure out how to solve this problem.

Now this idea is true in general when you have circular motion, you need the energy equation, which is the tangentially integrated Newton's second on the tangential direction.

And you need Newton's second law in the direction perpendicular to that circular motion.