

Consider a rod of mass  $m$ , and suppose I apply a force to this rod.

Let's say a force like that.

So we know that as a result of that applied force, the center of mass of the rod, which we can imagine is right at the center the rod, will translate with some acceleration, such that the vector  $f$  is equal to the mass of the rod times the acceleration of the center of mass.

Now recall that for a rigid body, this equation will be true regardless of where on the rod I apply the force.

So for example, if I draw the rod again over here, and I apply the same force, vector  $f$ , but I apply it, let's say, on the right hand side, I'll still have the same  $f$  equals  $ma$ .

All I specify with  $f$  is its magnitude and direction.

But for a rigid body, no matter where on the body I apply that force, the acceleration of the center of mass will be the same.

Now, we know from experience, however, that the motion of the rod is different if I push it at the center at one end or at the other end.

The point is, though, that the motion, the overall motion, the overall translation of the object doesn't just involve the translation of the center of mass, but it also involves some rotation, in general.

So in fact, there is a theorem called Chasles' theorem, which tells us that the most general displacement of a rigid body can be split into the translation of the center of mass and a rotation about the center of mass.

Our  $f$  equals  $ma$  relation tells us how an applied force affects the first part of the translation of the center of mass.

But how does that applied force affect the rotation of the rigid body?

We'll see that this depends not only on the magnitude and direction of the force, but also on where the force is applied, OK?

That's different than the translation of the center of mass, which only depends upon what the force is.

The rotation about the center of mass depends both upon the force itself and where on the object that it's applied.

In fact, in the next few lessons, we'll see that there's a special equation of motion for describing rotational motion.

It's analogous to Newton's Second Law of Motion, analogous to  $f$  equals  $ma$ , applying to rotational motion.

And we write that relationship.

Its rotational equation of motion is written as  $\tau$ , which is a vector, is equal to  $I$  times  $\alpha$ , which is a vector.

So  $\tau$  is a new quantity we haven't talked about before called the torque.

It's a vector.

$I$  is the moment of inertia that we've already encountered, and  $\alpha$  is the angular acceleration of the rotational motion.

Now,  $\tau$  is a sort of rotational analog to the force, and it depends on not just the force itself, but on where the force is applied.

Computing the torque,  $\tau$ , depends upon something called a vector product or a cross product.

It's a way of multiplying two vectors together to produce a third vector.

In that way, it's different than the dot product or scalar product that we discussed earlier, where we multiply two vectors together and get a scalar or a pure number.

This is a different operation, the vector product or cross-product.

Torque is the first physical quantity we've encountered, but it won't be the last, that involves a cross-product in its definition.

And so, before giving you a formal definition of torque, we'll first review the mathematics of cross-products, and we'll do that in the next lesson.

Now, the angular acceleration,  $\alpha$ , tells us about how the rotational motion changes.

Again, just like in  $f = ma$ , where we could divide the two sides of this equation into dynamics and kinematics.

Kinematics is telling us about a geometrical description of the motion, or the translation of motion, and the dynamics  $f$  is telling us about how the applied forces cause changes in the motion, changes in the kinematics.

Likewise, for rotational motion, the angular acceleration,  $\alpha$ , tells us about the geometry of the rotational motion, and specifically how the rotational motion is changing, and the torque is telling us about the application of forces and how that causes changes in the rotation of motion.

So that leaves one other quantity, which is the moment of inertia.

And I want to talk about what that means for just a moment.

So the term "inertia," the term "inertia" in physics represents a resistance to an applied force.

It tells us how difficult it is to change an object's motion.

So for example, suppose I have two blocks, one of mass  $m$  and another that's 10 times as massive.

So  $10m$ .

Newton's Second Law,  $f$  equals  $ma$ , tells us that if I want to accelerate the heavier mass to the same rate that I do with a smaller mass, I'll need to apply a force that's 10 times larger.

So for translation of motion, the mass  $m$  represents the notion of inertia, the resistance to a force.

It tells us how much force we have to apply to achieve a certain change in motion.

Now, for rotational motion, that role is played by the moment of inertia  $i$ .

That tells us how difficult it is to change the rotation of an object.

If I increase the moment of inertia by a factor of 10, then I'll need a torque that is 10 times larger in order to achieve the same change in rotational motion, the same angular acceleration.

But recall that our definition of moment of inertia for some rigid body is, if I break up the rigid body into a bunch of little pieces, and for each piece I take the product of the mass of that piece,  $\Delta m_j$ , times the perpendicular distance of that piece from the rotation axis-- I call that  $r_j$ -- and if I sum that over the entire object, the entire body, that gives me my moment of inertia.

Well, so-- sorry, this is  $r$  squared.

So it's the mass of each element times the distance of the axis squared,  $\Delta m_j$  times  $r_j$  squared.

Now, I can increase the moment of inertia by a factor of 10 by increasing the total mass of the object, but you can see from this equation that I can also do it with the same mass by changing the location of the mass.

If I increase the  $r$ 's, if I move the mass, the same mass, but I move it to be farther away from the rotation axis, that also achieves an increase in the moment of inertia.

So what that tells is that for rotational motion, it's not just the amount of mass that matters, but also how that mass is distributed.

OK, so in that sense, rotational inertia is different than translational inertia.

For translational motion, all that matters is the mass.

If I increase the mass by a factor of 10, then it will become a factor of 10 more difficult to get that object to accelerate.

I need a factor of 10 larger force.

But with rotational motion, I can increase the notion of inertia.

I can make it more difficult to change the rotation of motion either by changing the mass, by increasing it, or by making the distribution of mass be further away from the rotation axis.

So as an example, imagine if you were rolling a wheel up a hill.

It's different if I have a wheel whose mass is distributed evenly over the whole disk, or if I have all of the mass in the rim.

If all of the mass is in the rim, then from this equation, we see the moment of inertia is larger for the same amount of total mass.

And so it's much more difficult to roll a wheel up the hill if all of the mass is in the rim than it is if the mass is distributed evenly over the wheel.

So our rotational equivalent to Newton's Second Law is,  $\tau = I\alpha$ , the torque equals the moment of inertia times the angular acceleration.

In the next few lessons, we'll see how torque is defined, and we'll derive this expression for the dynamics of rotation of motion and see how to apply it.