

We would now like to apply the momentum principle to analyze the motion of the rocket.

So what is our momentum principle?

We know that the external force at time t is equal to the rate of change of the momentum of the system over t .

Now, recall we're going to actually use the formal definition of a derivative to write this as a limit as Δt goes to 0 of the momentum of the system at time $t + \Delta t$ minus the momentum of the system at time t divided by Δt .

So in order to analyse the rocket, what we need to do is separately analyze the momentum at our two states.

So we have a state at time t .

And we have a state at time $t + \Delta t$.

And our goal will be to analyze the system momentum at time t , and separately the system momentum and time $t + \Delta t$.

And then we can apply the momentum principle.

So let's begin by analyzing the system at time t .

So recall that at that time, we had our rocket and this is our time t .

And the rocket had a velocity V of r or t .

And what we identified the mass, m - r of t -- recall this was the mass-- the dry mass of the rocket and the mass of the fuel.

And so now, it's very simple to write down the momentum of our system at time t .

That's equal to just the mass m of r of t times the velocity of the rocket at time t .

And so we can use that in our expression for the momentum of the system.

Now our next step is to consider the system the time $t + \Delta t$.

Now recall, we still have the rocket but the mass of the rocket has changed.

That's inside the rocket, so this is dry mass of the rocket plus mass of the fuel, but recall some of the fuel has

been ejected outwards.

So how do we depict that in our momentum diagram?

Well let's just symbolize that by a certain amount and we're going to call this Δm -fuel.

Now, what we need to do, again, is to have our velocities.

So here, the velocity of the rocket is now the velocity at time $t + \Delta t$.

And what about the velocity of this fuel that's being ejected?

Well, in our problem the fuel is ejected at a velocity u relative to the rocket.

So what we have is u is the velocity of the fuel relative to the rocket.

But we've been choosing the ground frame as our reference frame.

And the rocket is moving at a velocity at time $t + \Delta t$ with respect to that ground frame.

And so recall that the velocity of the fuel in the ground frame is equal to the velocity relative to the rocket plus the speed of the rocket with respect to the ground frame.

And that's what we're going to draw on our diagram as the fuel.

And so now we can finish this analysis-- and we'll apply that next-- is to write the momentum of the system at time $t + \Delta t$.

Now this is going to have two terms.

It has the mass of the rocket at time $t + \Delta t$ times the velocity of the rocket.

And we also have to add mass of the fuel times the velocity of the fuel with respect to the ground.

And now we have the momentum at time $t + \Delta t$.

So we have both of our pieces here.

And we still have a little bit of analysis because we'll use our mass conservation equation and our relative velocity condition to simplify this expression.

So we'll do that next.