So I would like to tell you about friction today.

Friction is a very mysterious subject, because it has such simple rules at the macroscopic scale, and yet-- as I will show you and tell you a little bit about today-- very different rules at the nanoscopic scale.

So I would like to tell you about friction at the nano scale.

So at the macro scale, there are classic rules that were discovered very early on.

For macroscopic objects, it was actually Leonardo da Vinci who found the first laws of so-called "dry" friction.

So this was da Vinci as early as in the 15th century.

And basically, he discovered that friction depends on the force, on the weight of the object on the surface, and that there's a certain coefficient that depends on the material.

Da Vinci did not publish his notes.

They were just hidden in his notebooks for a long time.

And Amontons in the 17th century rediscovered the very same laws that da Vinci had found.

And these are two very simple laws.

The first law is that if you have an object sliding on a surface-- so there's a force that you are pulling on the object, and there's the friction force acting on it in the opposite direction-- then Amontons found that the friction force is proportional to the normal force.

So the normal force is the weight of the object.

So he found that the friction force as a function of normal force is just the simple interdependence.

So if you make the object twice as heavy, the friction force will be twice as large.

This makes, maybe, sense.

It agrees with our intuition that maybe an object twice as heavy should have a force twice as large.

The second law of friction is much more surprising-- namely, that if you have two objects of the same weight-- so you have the same normal force-- but very different areas of contact, then the friction force is the same.

So if you have a friction force for this object-- let me call it friction force one-- and the friction force of this object-- friction force two-- then the friction forces are the same.

One equals friction force two.

So surprisingly, the friction force is independent of contact area.

This seems very counter-intuitive, because if you think of friction as some kind of sticking, some kind of effect where two surfaces rub together, then we might naively assume that the larger the surface, the larger the friction force.

So one maybe intuition of why this might not be the case, or one possible explanation, is that surfaces always have roughness.

And so the actual contact area between two surfaces-- so if I really draw the surface of the object here-- then the actual contact area between two surfaces might not be the observed area of the macroscopic object.

And so this may be one way to explain why the friction is independent of the area, because it might only be proportional to the actual contact area of the system.

We have already described two of the laws of macroscopic friction.

There's a third law that was actually discovered by Coulomb.

And again, this is a little bit surprising result-- namely, that for this kind of friction, the dry friction, it is independent of the velocity of the object.

So this is, for instance, something that is not true for air resistance.

If you drive a car, the faster you go, the more friction there is, the more air resistance there is.

But if you have two surfaces sliding on top of each other, then Coulomb found that the friction force is independent of velocity.

Again, this is something quite counter-- not quite, but somewhat counter-intuitive.

And as I will show you a little bit later, it actually is no longer true when you look at the nano scale.

Coulomb was also the first one to distinguish static friction, which is the force with which an object resists motion for a while.

You make the force larger and larger, so for a while, the object will stick.

And then it will start sliding.

So he distinguished static friction from dynamic friction.

And because we have this law that says that the frictional force is proportional to the normal force, I will remind you of the first law-- the friction force being proportional to the normal force, like so-- we can define a slope.

So we can write the friction force as some slope, mu, times the normal force.

These are both forces, so they have the same units, which means that the so-called coefficient of friction-- which could be therefore static frictional or for dynamic friction, for kinetic friction-- this coefficient is just a number.

And typically, this number is between 0.3 and 0.6.

This makes a lot of sense.

The friction force is not quite as strong as the normal force.

So the friction force with which an object resists motion is not quite as large as the normal force with which it pushes down on a certain surface.

However, surprisingly, there are materials where mu is actually larger than 1.

It's possible, for instance, for rubber.

Which means that an object can be harder to pull sideways than actually-- it can be harder to pull an object sideways than to actually lift it from the surface.

So this is a surprising result.

Now, it turns out that despite these very simple laws, our fundamental understanding of friction is rather limited.

So for instance, nobody in the world can predict this coefficient mu of friction.

Nobody in the world can, from microscopic or principle say-- being told what the two surfaces are, say plastic or wood-- what is the friction coefficient.

Can you calculate it for me?

We cannot predict it.

We can only measure it, and then tabulate it-- make a table-- and say this kind of object on this kind of surface has this kind of coefficient for friction.

So that's one, if you want, failure of physics.

How is it possible that we can't describe, and quantitatively describe, something as simple as that?

Also, in many instances, we would like to change the coefficient to friction.

Typically, we would like to reduce friction, because friction is something where we dissipate energy.

It is estimated that as much as 3% of the nation's GDP is "wasted" on friction.

So friction destroys energy in the amount of billions and billions of dollars-- cars driving on streets, machines working, and so on.

Sometimes friction is, of course, desirable.

If you want to stop your car on the road, you want friction to actually work.

You wouldn't like to do away with it completely.

But in many instances, it's just a source of energy loss.

So if we could even make a small change in the friction, this would be not only important for science, but it might also be very important for technology.

So here, I've told you about friction of macroscopic objects.

Now people-- chemists, physicists, other scientists, engineers-- are able to build smaller and smaller machines.

And there's tremendous interest to build nanoscopic machines-- machines that are maybe only a few molecules or a small number of molecules or atoms wide and big.

And there's been tremendous success.

People have built nano pumps-- tiny pumps-- even a tiny car, consisting of a relatively small number of molecules that can move on the surface if you inject the electrons.

So as I would like now to tell you about how bad friction is at the nano scale.

So let's ask the following question-- how bad is friction at the nano scale?

Your general intuition might be that friction gets worse at the nano scale, simply because the bulk-- the volume of an object-- is what determines how many atoms are in it, whereas friction is a surface effect.

And because the volume increases more quickly with the size of the object then does the surface, you might think that probably, there's a bigger problem with friction at the nano scale than there is for macroscopic objects.

To illustrate how bad this really is, let's consider a tire of a car.

So this tire is rolling as your car is driving.

And we can try to estimate what happens to the wear of this tire.

So let's say that this tire maybe wears off.

Wear is on the order of maybe, let's say, 10 millimeters every, maybe, 50,000 kilometers.

So that means we take off a number of atomic layers here every 50,000 kilometers.

Now, how many atomic layers is 10 millimeter?

If we say that one atomic layer is on the order of, let's call it 1 Angstrom-- which is 10 to the minus 10 meters-- then 10 millimeters is 10 to the minus 2 meters.

So that is equal-- since one layer is 10 to minus 10 meters-- that is equal to 10 to the 8 atomic layers.

We can also ask ourself, how far has the tire been moving?

So if we assume that the circumference of the tire-- so this is the radius of the tire.

Let's guess that maybe the circumference of the tire here, which is given by 2 pi r, is on the order of 1 meter.

Then this means that 50,000 kilometers-- or 5 times 10 to the 7 meters, which is 50,000 kilometers-- is basically 5 times 10 to the 7 revolutions.

So in 50 million revolutions-- 5 times 10 to the 7 revolutions-- we lose 10 to the 8 atomic layers.

So it comes out that we lose one to two atomic layers per revolution.

If we had used maybe a size of an atom more closely as two Angstroms, we would come out at about one atomic layer.

So this is a remarkable effect.

Even when we drive our car, each time the tire rolls around, we leave one atomic layer on the street.

For a macroscopic object, as we see, it doesn't matter.

We can drive a very large macroscopic distance before we wear off a few millimeters of the tire.

But if the tire of the car was itself only a few atomic layers big, then you see we would have a huge problem.

So this is why people are interested in understanding, and maybe manipulating, friction at the nano scale.

So what does friction at the nano scale really look like?

And why are people interested in it after so many centuries of first studying friction?

One reason is that now one can do experiments at the nano scale.

And the new tool that has enabled that is what is called an atomic force microscope.

What is an atomic force microscope?

Well, it's essentially a very, very sharp tip-- atomically sharp tip.

So you have a surface here, which consists of atoms.

And these are individual atoms.

And you're trying to study the friction.

And now what you do is, you bring an object here shaped in the form of the tip, which also consists of atoms.

And the object is so sharp that you have, ideally, just one atom near the surface-- one or more atoms near the surface.

So this is what an atomic force microscope is.

You can pull on this with some force.

And then you can try to measure the force of resistance-- the friction force that is due to the interaction between the atom near the surface and the atoms forming the surface.

So we might call this the substrate.

And this would be the moving object.

And we can either study static friction by pulling the objects, but not with a force large enough to move, or maybe we can apply a larger force so the object starts moving at a certain velocity v, and we can measure friction.

Now this, as simple as it may seem, is still too complicated for physicists.

So we try to make the model even simpler.

So how do physicists do this?

Well, they say that the surface still consists of atoms, but now we're mostly interested what happens to this tip.

We're going to say, well, somehow, what matters is that this atom is bound to the tip.

So maybe we can model it in the following way.

We have here the macroscopic part of the tip.

And then we're going to say that the surface atom is really bound in some way to the tip.

And the simplest way that we can imagine the surface atom to be bound to the tip is via some spring with some specific spring constant.

So basically, the one atom that makes the contact with the surface is bound via spring to the rest of a tip that is moving-- to the rest of the moving object.

So this is how we view the tip as physicists, to make a model.

And then how do we view the surface to make a model?

Well, we say this is really a periodic arrangement of the atoms.

So maybe it makes sense to kind of assume that the associated potential is also periodic at the atomic scale.

So this is the potential as a function of position.

So basically, we model friction as spring plus periodic potential.

This is a very simple model.

And it was first introduced now about 90 years ago.

So by Prandtl-- by a German physicist called Prandtl-- and independently by another physicist called Tomlinson.

And this is called the Prandtl-Tomlinson model.

And they discovered it, or introduced it, in 1928 and 1929.

And it turns out that this very simple model of a spring and the periodic potential captures much of the essence of nano friction.

So as physicists, now we have a spring and the periodic potential.

How do we think about this?

Well, one way is to think about the energy in the system.

So we can plot the energy as a function of position.

And now we have our periodic potential, like so.

And how do we model the spring?

Well, this is going to be the potential energy, v. How do we model the spring?

Well, the spring has a linear force.

The force is proportional to the displacement, which means the potential is quadratic in this placement.

So this is the potential for spring.

This is what the spring does, and this is what the substrate does.

And what we need to do is, we need to add these two together.

So what that means qualitatively is that the total potential of the system might look something like this.

So as we now translate the spring across the surface with some velocity, then you can see that this addition between the fixed substrate potential-- this one is fixed-- and this spring potential is moving.

You can see that this will lead to a time-varying potential for the object.

So let's look now, in a simulation, what that time variation might look like.

So what you see here is a combination of a spring and the periodic potential.

And the periodic potential has been chosen a slightly different strength than I'm showing you here.

It has been chosen weaker so that instead of many minima in this total potential, there are only two minima.

And what you see happening in this system is that, as the particle is moving, the spring tries to pull it across a maximum of the periodic potential.

At some point, the atom is released, and it releases energy that is taken up as heat by the substrate.

And then, the object is pulled towards the next minimum.

The minimum disappears slowly.

The object is released again, et cetera.

So basically, in this model, we can understand friction as the external force pulling the object over successive maxima.

So that's nice.

In this model we can understand why there is heat generated.

There's always heat generated when the atom loses this extra energy, because it is stuck for a while in a minimum of the potential, which is not the absolute minimum of the potential.

And then the moment it's released, it releases kinetic energy that is converted into heat.

Now we can have a different situation.

So this was for strong or moderately strong potential.

So basically, this was a situation where there are just two minima in the potential, like so.

We can make the potential a little bit weaker.

And in that case, the curvature of this potential-- of the periodic potential-- the curvature in one direction might not be enough to overcome the opposite curvature of the spring.

So in this case, we can end up with a potential just slightly distorted, but we have only one minimum.

So in the next movie, I will show you what happens when we have just one minimum.

In that case, you can see that, because there is no second minimum in the system, the object-- in this case, the tip-- follows quite smoothly the minimum of the potential.

And no energy is released.

No heat is released in the problem.

This model-- this Prandtl-Tomlinson model-- is interesting.

If in this model, we plot the friction force versus the corrugation of the potential corrugation-- so we'll call this the corrugation of the potential, u, which is proportional to the normal force for a macroscopic object-- then what we find is, yes, we find a linear dependence.

So this is, if you want, the corrugation or the normal force.

What we find is a linear dependence, but only above a certain critical value.

So the friction force actually looks like this.

It's 0 until the potential becomes strong enough.

Basically, you can think of this curvature becoming larger than that curvature.

And then, the friction force sets in.

So this is at the nano scale.

And our simple macroscopic friction law would have predicted something of the same slope at the macro scale-- something of the same slope with a [INAUDIBLE] offset.

So we can kind of see that, at the nano scale, the friction is a little bit more complicated.

There's a region where there's no friction whatsoever.

But then it increases linearly with the normal force.

So you can imagine that if I go to macroscopic normal loads, then the difference between these curves, at least fractionally, will be quite small.

The difference between these curves, at least fractionally, will be quite small.

And so we can see how the law of the nano scale, explained by this very simple Prandtl-Tomlinson model, approaches the law at the macro scale.

So far, I have told you about something very simple-- namely when the contact is a single atom.

And even making this very simple approximation, we can already understand why the friction force is approximately proportional to the normal force.

Now in real life, probably there is more than one atom touching the surface.

So let's consider a contact area where several atoms-- maybe a long chain, maybe just a few atoms, in the case of nanoscale probably just a few atoms-- several atoms will be making up the contact.

So instead of considering this situation with one atom on a spring, which is the single atom case, now let's consider a situation where several atoms make up the contact.

So again, here we have the substrate with our periodic potential that is ultimately coming from the individual atoms.

But now we will consider more than one atom making up the surface.

So in this case, how do we model the system?

Well, we think that these atoms are still connected by springs to the macroscopic object.

But typically, these atoms will also have forces between them.

So this is my physicist's model of what happens when more than one atom touches the surface.

Now these are only masses of springs and some simple periodic potential, and yet the situation is quite complex, and in many ways, counter-intuitive.

And the first thing that changes compared to the single atom is that now I have the distance between the atoms as a parameter.

So if I label the period of the periodic potential as a, and maybe the period of my object of the distance between the atoms in the object as d, then I can have different situations.

So one very simple case is when d is equal to a-- when the two periods are the same-- or in general, when d is a

multiple integer of a, and n is an integer.

So let's see what happens in this case.

Very naively, all the atoms are doing the same thing relative to the surface.

So you might expect them, maybe, to move in exactly the same way.

They will all move together.

And these springs between them will not stretch.

In this case, I can forget about the springs between the atoms in this simple case that we'll call commensurate.

Basically, when the period of the object matches the period of the substrate-- the commensurate case-- then the atoms are at equivalent positions throughout the substrate period, the lattice of the springs between them will not stretch.

And it's as if they are not there.

So in the commensurate case, what you can see here is, as the atoms are pulled across the periodic potential, because of this commensurability condition where d is equal to a or a multiple integer of a-- basically, two periods are matched-- all the atoms are doing the same thing at the same time.

So they get pulled, pulled, pulled.

They first stick, and then they all slip at the same time.

And you can imagine-- and you can kind of see visually in this case-- that the friction is the same as the single atom friction multiplied simply by n.

So this is just the single atom friction that we have seen before multiplied by just the number of atoms that makes up the contact area.

Now there's a different case, which we might call incommensurate, where d is not equal to a, and d is not an integer multiple of a.

So maybe d could be 1.5 a, or 2/3 of a, or some other number.

And what is the most incommensurate case that we can imagine?

Well, the most incommensurate case for d and a is that the ratio of d and a is an irrational number.

So an irrational number would be something like square root of 2, or for our purposes, it turns out that the most irrational number in a mathematical sense is 1/2 squared of 5 plus 1.

This is the so-called Golden Ratio.

The ancient Greeks believed that it had magical properties.

For instance, when they built the temples, the two sides of the temples-- the two sides of the rectangle-- were related by this very strange number, the so-called Golden Ratio.

It is assumed to be aesthetically very pleasing.

Now mathematically speaking, the Golden Ratio is very interesting, because it is the most irrational number, in some sense, that you can devise.

Now we can choose such an incommensurate ratio of d over a, and see what happens in this case.

So you can see here that, in the case of an incommensurate ratio, the behavior of the transport-- the behavior of the motion of the object-- changes dramatically.

Instead of all the atoms sticking and slipping together, like we had in the commensurate case, now the atoms move one by one.

Basically, the first atom moves over the barrier, the spring stretches, which facilitates the motion of the next atom over the barrier, the next atom, and so on.

So these atom chains move like a caterpillar.

They essentially move one by one.

In technical language, we call these things kinks.

You can see that they are periodic compressions.

They are compressions in the chain and stretches in the chain.

And so these atoms move like a caterpillar.

And now an interesting fact is that in this case, friction is dramatically reduced.

And depending on which regime you are, the friction can even disappear altogether at the nano scale.

The reduction was so large that some people have even coined this with the word superlubricity.

Interestingly enough, even though this is a very simple system, it's just, if you want, balls and springs, this superlubricity was only discovered in the late 1980s, even though we know all the physics for more than a century.

So you can see just a tiny, tiny rearrangement of the atoms-- not to be any more commensurate with the lattice, but to be incommensurate in terms of an irrational number-- can change friction properties dramatically.

In fact, there was a French scientist called Aubry that first pointed out that there's a very interesting transition that happens in this system of a periodic potential and atoms connected with springs when you choose, as the ratio between these two length scales, the Golden Ratio.

And this is called the Aubry Transition.

So far, we have considered only, if you want, the mechanics of the motion, which is equivalent to saying that we have assumed that the system is at temperature T equal 0.

Basically, the atoms remains at the minimum.

It doesn't wiggle around because of temperature, and we have derived the friction in this limit.

Now what happens for finite temperature-- by finite, meaning a temperature that is larger than 0.

Well, if you think back of our simple single-atom model of friction, where maybe an atom is stuck here as the potential is moving with some velocity v, then in the 0 temperature limit, this atom would only be released at the time when this minimum actually disappears.

And then it would have a high energy and it would dissipate this energy to be cooled to the next minimum.

However, if we have a finite temperature T-- some temperature scale T-- that means that the energy of the atom in the potential is not 0, the kinetic energy.

But the atom has a smeared-out kinetic energy and potential energy that is like so.

So what that means is that the atom can [INAUDIBLE] hop over the barrier and find the new minimum, without actually having to be pulled over this maximum.

So what we expect is that temperature effect might reduce friction.

So people call this thermolubricity-- the effect that when you heat up the surface-- and in many instances, you would have to substantially heat up the surface-- then friction is reduced, because the atom can find the new minimum of the potential without actually having to be pulled over the barrier.

So let's see in a small simulation what that might look like.

What you see here is now that the atom has a chance of hopping between the two minima, back and forth.

And it has a certain probability to be found at either one of the two minima, which is indicated by the size of the red circle, in this case.

So you can see that when temperature is present, then the atom can follow the minimum locally-- the absolute global minimum-- simply because it can hop back and forth between the barrier without actually experiencing much friction.

So in particular in the limit when the temperature is very, very high, or the velocity of the atom is very, very low, then the atom with hop many times back and forth between the two minima.

And the distribution between the two minima would be simply given by the Maxwell-Boltzmann distribution, which means that the atom will be predominantly found in the global minimum.

And they'll just be following the potential along and the friction in this system will be quite small.

So in this case, if we do a measurement of friction versus velocity in a simulator-- that we do, then this is what the result might look like, or what the result does look like.

We see here different ranges of friction.

We see a range where the friction does not depend on velocity at all.

Friction is 0.

Then there's a range where the friction increases with velocity, but only very weakly logarithmically.

Please notice that this is a logarithmic scale for the velocity.

So the velocity here changes over five orders of magnitude.

So friction increases with increasing velocity.

Then there's a range of frictions actually independent of velocity, just like the simple macroscopic law would

predict.

And then it turns out when you move the atom very, very fast, then it basically doesn't have time to dissipate all the energy.

And friction is reduced again, effectively, because the atom is hotter.

So we can see that the simple macroscopic law of friction only applies in a region, which in this experiment was relatively narrow, of relatively low temperatures.

So let's summarize what happens to the loss at the macro scale.

So the first law at the macro scale was that the friction force was proportional to the normal force.

And we found that at the nano scale, actually what happens is a displaced curve, where there is no friction in a certain region, and then the friction follows parallel to the macro results.

So this would be the actual friction at the nano scale.

And we see that it's a fairly good approximation to the macroscopic friction, but with a small offset.

Our second law of friction was that friction is independent of surface area, of contact area-- basically, the idea that a high object and a flat object of the same mass experience the same friction.

And we see that at the nano scale, it's replaced by a much more complicated law that depends on the arrangement of the atoms, or on, if you want, commensurability.

so we have non-equivalent arrangements of the atoms that can either lead to large or small friction.

And somehow, the macroscopic law is some kind of average over these behaviors, if we allow for randomness in the positions of the surfaces.

And finally, our third law-- that at the macroscopic scale, friction is independent of velocity-- we found at the nano scale that this can be true, but it is generally true only in some finite temperature range-- namely, that if you move the object very, very slowly, then thermal excitations allow you to always find the global minimum, and friction disappears-- the effect called thermolubricity.

So one can see that there are nice connections between the nano scale and the macro scale.

But many open questions remain, in particular, concerning the point two, which is that the friction is independent of the contact area.

What does the contact area really look like for two macroscopic objects?

And how is it that this independence on the surface area-- or at least on the apparent contact area-- actually arises from the properties of friction at the nano scale?