

We now would like to apply our energy momentum rule and momentum to analyze a one-dimensional elastic collision with no external forces.

Let's remind ourselves, we'll call it the energy momentum equation said that V relative initial was equal to V relative final.

So we have V_{1x} initial minus V_{2x} initial-- that's the x component of initial relative velocity-- is equal to the final x component of the relative velocity.

And that was our energy momentum law.

Now the momentum condition that it's constant was our equation V_{1x} initial plus $m_2 V_{2x}$ initial equals $m_1 V_{1x}$ final plus $m_2 V_{2x}$ final.

Now let's see how this linear system is much, much easier to solve.

Let's look at the same problem that we solved before where m_2 was equal $2m_1$.

And also, we were in the laboratory frame, so V_{2x} initial is 0.

And that tells us that the initial velocity, relative velocity, is simply the velocity of object 1.

So let's just write our two equations down again and see how much simpler our system is.

V_{1x} initial is minus V_{1x} final plus V_{2x} final.

So we have minus plus.

And our momentum condition, remember V_{2x} initial is 0.

The m_1 and 1, m_2 will be $2m_1$.

So we can cancel our m_1 s.

And we get V_{1x} initial equals V_{1x} final.

And m_2 is twice m_1 , so there's a factor plus $2V_{2x}$ final.

Now I want to solve for our target variable.

I look at these two equations.

I can see almost immediately that if I add these two equations, V_{1x} initial will cancel.

And I get very simply by adding, we get $2V_{1x}$ initial.

And this is $3V_{2x}$ final.

Or V_{2x} final is $\frac{2}{3} V_{1x}$ initial.

And let's just call this equation 1 and 2.

So we added.

And now to find V_{1x} final, let's see what we'll do there.

So we can do this a variety of different ways.

I think the simplest thing here to do, we could eliminate V_{2x} final by multiplying through by minus 2.

Or we can simply substitute in V_{2x} final right here.

And we get-- so let's do that.

Let's substitute that in right there.

And we get V_{1x} initial equals V_{1x} final plus 2 times $\frac{2}{3} V_{1x}$ initial.

When we bring that over to the other side, $1 - \frac{4}{3}$ is minus $\frac{1}{3} V_{1x}$ initial equals V_{1x} final.

And at the cost of introducing a new concept, we've found the algebra much, much simpler to solve in this problem.

And we can just double check our result that the initial velocity, relative velocity, was simply V_{x1} .

And the final relative velocity, V relative final, is minus V_{1x} final minus-- let's see, the final relative velocity is V_{1x} final V_{2x} final i hat.

And when we put that in, we have minus $\frac{1}{3} V_{1x}$ initial minus $\frac{2}{3} V_{1x}$ initial i hat.

And we have minus V_{1x} initial i hat, which is minus V relative initial.

And so we see that the relative velocity simply changed direction.

This approach is much, much easier.

And keep in mind that the energy momentum and the momentum laws are just rewriting our two fundamental constants of motion, kinetic energy and momentum.