

Now let's go back and analyze that same problem that we were looking at before of a wheel rolling down an incline plane.

And it's rolling without slipping.

But instead of using the energy method, we're now going to use the torque method.

So let's consider the center of mass.

And what we want to do is draw the forces.

We have a normal force.

We have gravity.

And we have the friction force about the center of mass.

Our wheel has a radius R .

Let's choose a coordinate system \hat{i} , \hat{j} .

And so we have a right-handed system \hat{k} , and that will correspond to some angle θ .

Now here we're now going to enlarge how we apply both translation and rotation.

And the beauty of this problem is we now can decompose our motion into translational motion and rotational motion.

So for the translational motion, we'll apply Newton's second law.

Now, if this is the angle ϕ , then that's the angle ϕ as well.

And so our forces in the \hat{i} direction, we have $mg \sin \phi$ minus the friction force.

And that's equal to the x component of the acceleration.

Now we also can choose the center of mass to calculate the torque.

And so what we're really just studying here is simply our old problem in the center of mass frame of fixed axis rotation.

And you can see gravity is acting at the center of mass.

So it produces no torque about the center of mass.

The normal force is directed towards the center of mass.

And so when we take that vector product of \mathbf{R} cross \mathbf{n} from cm to this point down here, the contact point, these forces are anti-parallel.

So the normal force produces no torque.

And the only torque that we have is from the friction force, and that friction torque is going to give us a positive angular acceleration in the \hat{k} direction.

It's at right angles with the vector \mathbf{R} .

So we have f_s times R equals I center of mass times α .

And these are our two dynamic equations.

But remember, when the object is rolling without slipping, let's just remind ourselves that V_{cm} equals R ω .

And if I differentiate, the A_{cm} which is what we're calling a_x , is equal to R α .

So this a_x here is the acceleration of the center of mass.

And that's our third condition.

And so now I see that I have three equations and my unknowns-- f_s , a_x and α .

And so I'm going to solve these equations for a .

And I'll look at these equations.

And what I'll do is I'll just substitute for α a_x over R .

And then solve this equation for f_s , and put it in there.

And so I get $mg \sin \phi$.

Now my f_s is equal to I_{cm} over R times α .

But α is a_x over R . So that's a_x and an R squared.

Notice dimensionally, I is mr^2 .

So this is just ma , the dimensions of force, mg dimensions of force, and that's equal to $m a_x$.

And now I can solve for a_x .

And I get $mg \sin \phi$ divided by m plus I_{cm} over R^2 .

Now a_x is a constant.

And we can, from our kinematic equations, if our object is moving a distance s as it drops height h , we know from kinematics and we can work this out.

We have that X_{cm} is one half $A_{cm} t^2$.

And we know that the velocity V_{cm} equals $A_{cm} t$.

And so when we put these two together, this is the distance s , we get that the velocity v_{cm} is equal to the square root of $2s$ times A_{cm} .

Now s is equal to h over $\sin \phi$.

$s \sin \phi$ is h .

And so the V_{cm} equals the square root of $2h \sin \phi$ times A_{cm} -- but we've solved for A_{cm} -- $mg \sin \phi$ over m plus I_{cm} over R^2 .

And so we get the square root of $2mgh$ over m plus I_{cm} divided by R^2 .

And this agrees with our energy method.