Recall, when we were examining the motion an object, in two dimensions, we introduced Cartesian coordinates and a position vector.

Now let's suppose the object has moved to a new point, along the orbit.

Well, we'll write another vector r of t.

And let's say this took a time delta t to the new point.

And what we want to define is the displacement of that object.

So that's a vector delta r.

And recall that a vector of time of t plus delta t is equal to the old vector r of t plus this displacement vector delta r.

Now what we want to consider is a limit as delta t goes to 0.

And let's just look graphically at what that means.

As we move this delta, delta t-- as delta t gets smaller and smaller and our object is getting closer and closer to its position at time t, and the position vector r of t plus delta t is getting closer and closer to r of t delta t, the key fact is that if we do a tangent to the orbit, then the limit of delta r is approaching tangent to the curve.

So in the limit, delta r, the direction is tangent to the orbit.

So that's our first key property of delta r.

Now the second thing we want to express is, if we write delta r, as a displacement in the i-hat direction and a displacement in the j-hat direction, now again, maybe we can just clean this up a little bit, and see what we mean by that.

So here's our delta r.

And we have a little delta x in this direction, delta y in that direction.

Remember delta x or delta y can be positive or negative.

That's all right.

Now if we want to define our velocity as the limit, as delta t goes to 0 of delta r over delta t, then what we see is we have two pieces, the limit as delta t goes to 0, of delta x over delta t i-hat, plus the limit as delta t goes to 0 of delta y delta t j-hat.

And the definition of these limits, we'll write that as the derivative dr, dt.

So the velocity is dr, dt.

And that's equal to dx, dt, how that coordinate function is changing in time, i-hat plus dy, dt j-hat.

Now as far as notation goes, we write this philosophy as an x component of the velocity plus a y component of the velocity, where the x component, the x, is dx, dt.

And the y component is dy, dt.

Now recall that the direction was tangent to the curve, but the magnitude of the velocity, what we call the speed, is just the sum of the squares of the components, the square root.

And so now we've describe what we refer to as the instantaneous velocity.

So far we've looked that a trajectory in two dimensions.

Let's again consider some type of motion where we choose a positive y-axis, a positive x-axis, an origin, e at vectors, i-hat and j-hat.

And I'll have some type of trajectory, where our object is moving like that.

We know that at this particular time, the velocity is tangent to this trajectory, at that point.

And now, what we'd like to do, is try to describe-- we've described it's two components ex and vy as a vector.

So if you did vector decomposition, you would write a vector like this and a vector like that.

This is the x component.

That's the y component.

And now if I define this angle theta, we know that a vector has a direction and a magnitude.

We've seen what we call the magnitude the speed.

So that's just the sum of these components squared, square root.

Speed is always positive.

So we always take the positive square root.

And now what about the direction of this vector in the xy plane?

Well, we can see from our geometry that the tangent theta is given by the y component over the x component.

Or one could say that the angle theta, at this given time, is the inverse function of vy over vx.

And so now we've described not only the direction of velocity, but the angle that it's making with the horizontal axis.

And so we have now completely described the velocity, instantaneous velocity, vector at time t in terms of its two component functions, its speed and the angle that makes at the positive x-axis.