We've now described what we call the average velocity for a time interval between when the runner started at time t to a later time at t plus delta t.

And we described that as the component of the displacement vector divided by the time interval times a unit vector, i hat.

Now what we'd like to ask is a separate question.

So our question now is, what do we mean by the velocity at some specific time, T1?

Now in order to understand that, let's just make a plot of the position function.

So remember we called the component of the position function x of t.

So we're going to plot the component of the position function with respect to time.

Now let's just say that the runner started at the origin of time equal 0.

So I can make some type of arbitrary plot of that position function.

And let's indicate in particular, the time T1.

So what this represents is x of T1.

And so first I'd like to consider the interval T1 and T1 plus some later time, delta T.

So let's make this T1 plus delta t.

This is the time delta t and up here we have our position function at T1 plus delta T.

Then for this time interval, the average velocity, so for this particular time interval, the average represents delta x over delta t.

So it's just rise over run.

It's just the slope of this straight line.

So for this particular interval, the average is the slope of the line shown here on the figure.

Now this is just an average velocity and now what we would like to do is shrink down our interval delta T. So now let's make another case where we shrink delta t and let's again calculate the average velocity.

So for instance, suppose we have a smaller delta t and we draw that line.

Then our average velocity represents that slope.

And again, we keep on taking a limit.

So now we have another slope so we have one slope, two slopes, and now we shrink again to a new delta t and you can see that the slope is changing.

And if we consider the limit as delta t goes to 0 of this sequence of slopes, then what are we getting, you can see graphically, that eventually we will get to a line which is the slope of the tangent line at time T1.

And so in this particular case, what we mean by the instantaneous velocity, v at time T1, is the limit as delta x goes to 0 i hat here where we're taking, this is the limit, delta t goes to 0 x at T1 plus delta t minus x1 of t divided by delta t and the whole thing is a vector, i hat.

So what a limit is, is a sequence of numbers.

So we take a fixed delta t, we calculate the slope.

We take a smaller delta t, calculate the slope.

And each time we do that, the slopes represent a sequence of numbers and the limit of that sequence you can see graphically, is the slope of the tangent line at time t 1.

And so what we say is, v of T1 is the instantaneous velocity at time t equals t1.

And that's how we describe instantaneous velocity at some specific time.

If we were now being a little bit more general, we could just say that v at any time t is the limit delta t goes to 0 delta x over delta t ball direction i hat, and the only thing here is we're no longer considering T1 but an arbitrary time t.

This quantity, the limit, is awkward to write every time.

It has a name.

And that's precisely what we call the derivative of the position function.

So our instantaneous velocity is the time derivative of the position function at any instant in time.