

So let's talk about kinetic energy again.

Remember, when we have an object moving with speed  $v$ , we define the kinetic energy of the object as a scalar quantity  $\frac{1}{2} m v^2$ .

This is scalar.

And what's very important to realize, it is reference frame dependent because the velocity of the object depends on what reference frame we're in.

One thing to keep in mind is remember our definition of scalar product,  $\mathbf{a} \cdot \mathbf{b}$  is the magnitude of  $\mathbf{a}$ , the magnitude of  $\mathbf{b}$ , times the cosine of  $\theta$ .

So a vector dotted into itself is just the magnitude of  $\mathbf{a}$  squared, which we can just write as  $a^2$  because the angle is zero.

So here, we have  $\mathbf{v} \cdot \mathbf{v}$ .

So the dot product  $\mathbf{v} \cdot \mathbf{v}$  squared, enables us to write kinetic energy as  $\frac{1}{2} m \mathbf{v} \cdot \mathbf{v}$ . And we see our first example of the scalar product describing a physical quantity.

If we had a coordinate system where we wrote  $\mathbf{v}$  as  $v_x \hat{i} + v_y \hat{j} + v_z \hat{k}$ , and we took the dot product of that vector with itself, then we just know that it's the square of the components.

And so, in a Cartesian system, we can now say that the kinetic energy is  $\frac{1}{2} m (v_x^2 + v_y^2 + v_z^2)$  because that's the velocity squared.

So there's a simple way to express, abstractly, the kinetic energy as a dot product and explicitly in a coordinate system as the sum of the squares of the components of the velocity.