

We've defined moment of inertia of a rigid body already.

And we often are interested in it, because when we rotate a body about an axis, say an axis that's passing through the center of mass.

Then our kinetic energy depends in that moment of inertia through that axis.

But we may also want to consider rotation about another axis.

So suppose we consider an axis passing through close to the end and we rotate the body around this axis.

You can see from overhead like that.

Then we want to now consider how the mass is distributed about an axis that was parallel to the axis passing through the center of mass.

So we'll now make a little calculation.

But first, we want to quote the theorem.

So let's just draw an example of our rigid body.

And let's take the center of mass, and let's consider an axis that's going perpendicular to the center of mass.

We can think of the object is rotating about that axis.

And now let's consider another axis passing through a different point, but also parallel to this axis separated by a distance d .

Then the result that we want is at the moment of inertia about an axis passing perpendicular to the plane of the object through the axis s is equal to the moment of inertia about an axis passing through the center of mass-- notice these are parallel axes-- plus the mass of the object times the distance between the two parallel axes.

And this is a result that is very useful when calculating moments of inertia.

Of course, we could calculate the moment about any axis we wanted.

And we'll see that in a moment.

For example, we know that for a rod of length L that the moment of inertia through the center of mass was $1/12$.

And let's call the length of the rod L . So we have a length L .

And let's assume it's a uniform object.

And we'll calculate a moment of inertia through an axis through the end.

And so in this case, d is equal to L over 2.

And so I about the end axis is $\frac{1}{12} mL^2$ plus the mass times L over 2 square and a 12 plus a quarter is $\frac{1}{3} mL^2$.

And that means that all you need to know is the moment through the center of mass, and you can calculate the moment through any other axis.

Very useful theorem called the parallel axis theorem.

Now as I said, we can calculate the moment s .

And just to show you very quickly, if we pick s here, and we pick our little dm there, and we have a distance x , then the moment about s -- and let's say that dm is equal to the total mass divided by the length times some little distance dx -- then the moment is dm times x squared, where x we'll give this an integration variable x prime goes from 0 to x prime equals L .

And so we have for our dm m over L dx prime.

Remember, that's our integration variable.

And we have x squared.

So that's x prime squared.

And this is just the integral of x cubed over 3.

So we have $\frac{1}{3} m$ over L x prime cubed evaluated from 0 to L , and that comes out to $\frac{1}{3} mL^2$.

And that's in agreement with the parallel axis theorem.

And that's an example of parallel axis theorems.

You can do the same thing with the disk or other objects.