

When we analyzed how the position vector changed, we know that the velocity for circular motion is given by the radius times the rate that the angle is changing.

And it points tangential to the circle.

So let's draw a few characteristic arrows to show that.

At this point, we'll draw these pictures with $d\theta/dt$ positive.

So the velocity points like that.

It points like this.

It points like that.

And these are all the velocity vectors at different times.

Notice that if we make-- consider the special case in which $d\theta/dt$ is a constant, in that instance, the magnitude of the velocity, v , is given by r magnitude of $d\theta/dt$.

And that is also a constant.

But the velocity vector is changing direction.

And we know by definition that the acceleration is the derivative of velocity.

And so what we see here is where we have a vector that's constant in magnitude but changing direction.

And we now want to calculate the derivative in this special case.

We refer to this case as uniform circular motion.

So this special case is often called uniform circular motion.

OK.

How do we calculate the derivative of the velocity?

Well, recall that the velocity vector, $r d\theta/dt$ -- those are all constants-- because it's in the $\hat{\theta}$ direction, once again, will decompose $\hat{\theta}$ into its Cartesian components.

You see it has a minus \hat{i} component and a plus \hat{j} component.

The \hat{i} component is opposite the angle.

So we have minus sine theta of \hat{i} plus cosine theta of \hat{j} .

So when I differentiate the velocity in time, this piece is constant, so I'm only again applying the chain rule to these two functions.

So I have $r, d\theta/dt$.

And I differentiate sine.

I get cosine with a minus sign.

So I have minus cosine theta.

I'll keep the function of t , just so that you can see that-- $d\theta/dt \hat{i}$.

Over here, the derivative of cosine is minus sine $d\theta/dt$.

That's the chain rule-- sign of $d\theta/dt, d\theta/dt, \hat{j}$.

And now I have this common $d\theta/dt$ term, and I can pull it out.

And I'll square it.

Now whether do you think that dt is positive or negative, the square is always positive, so this quantity is always positive.

And inside I have-- I'm also going to pull the minus sign out.

And I have cosine theta of \hat{i} plus sine theta of \hat{j} .

Now what we have here is the unit vector $\hat{r}(t)$.

\hat{r} has a cosine adjacent in the \hat{i} direction and a sine component in the \hat{j} direction.

So our acceleration--