

Let's apply the work energy theorem principle to the motion of a block sliding down an inclined plane.

And here is an inclined plane at an angle, θ .

And let's choose a coordinate system.

We'll choose x equals 0 up here, or \hat{i} here.

And here is our coordinate function.

And suppose that the object starts at x_i , that it ends at x_{final} .

If we want to calculate the work energy then what we're going to learn is that this theorem is two different sides.

Calculating the change in kinetic energy is simply a description.

This is $\frac{1}{2} M v_{\text{final}}^2$ minus $\frac{1}{2} v_{\text{initial}}^2$.

That's a property, parameters of the system, at the initial state, the speed, or the velocity's speed, which is speed squared.

And the same property v_{final} in the final state.

Now over here we have two types of forces acting on this object.

So here we need a free-body force diagram first.

And this side is where the physics [INAUDIBLE] lie.

And so we want to draw our force diagram.

So if we have our block, we have the friction force, we have the normal force, we have mg .

Recall, if the plane is inclined θ that's also the angle θ .

If we chose \hat{i} , \hat{j} unit vectors, I just want to repeat that on my free-body diagram.

Now we can think of this integral as just one-dimensional motion in the x direction.

And so we have two different forces that we have to calculate.

The friction force is in the minus x direction so we're integrating minus the friction force with respect to

displacement from x_{initial} to x_{final} .

And what about the gravitational force?

Well the gravitational force has a component, $mg \sin \theta$ in the x direction.

So when we integrate that x component we have now plus because it's in the same direction.

Remember, we're displacing a little bit dx down the inclined plane so we're going from x_{initial} to x_{final} of $mg \sin \theta$, which is a constant, dx .

And so when you're applying the work energy theorem you need to integrate your forces and actually calculate the work.

Now again, if you looked in the \hat{j} direction, and we applied Newton's second law, $n - mg \cos \theta = 0$, and our rule for friction is $\mu_k n$ or $\mu_k mg \cos \theta$, then what we have is in both instances we have a constant force.

So it's just force times displacement.

So we have $-\mu_k mg \cos \theta$ times the displacement, which is $x_{\text{final}} - x_{\text{initial}}$.

Over here we have $mg \sin \theta$ times $x_{\text{final}} - x_{\text{initial}}$.

And now we've calculated separately both sides of our work kinetic energy principle.

As in all our physical laws the equal sign means the work is equal to the change in kinetic energy.

I'll emphasize that by now placing the equal sign because of our physical law.

And so equating $\frac{1}{2} M v_{\text{final}}^2 - \frac{1}{2} M v_{\text{initial}}^2$.

And now I have a relationship between the parameters of the initial state, which I'm calling x_{initial} and v_{initial} , and the parameters that describe the final state, x_{final} , v_{final} .

And depending on which set of these parameters are given I can conceivably solve for the other ones.

One thing I do want to point out when we do this example is we've described work as a dot product from A to B. Take the friction force.

Well in this instance, if we wrote this out explicitly it would be $-\mathbf{f}_k \cdot d\mathbf{x}$ from the initial to the final.

$\hat{i} \cdot \hat{i}$ is 1.

And so you see, we recover from x initial to x final of $f_k dx$.

And that was the first piece.

The second piece, the gravitational force, dotted into ds from x initial to x final.

Well, if you wrote down the gravitational force, mg , as a \hat{i} component and a negative $mg \cos \theta$ \hat{j} component, then when we take the dot product again where our ds -- here, we'll write ds as $dx \hat{i}$ -- then when you dot product $mg \cdot ds$, $mg \cdot ds$, we have $\hat{i} \cdot \hat{i}$, which is one.

But $\hat{j} \cdot \hat{i}$, they're perpendicular so that's 0, so the only piece that survives in the scalar product $mg \cdot ds$ is these two pieces.

And so we get the integral from x initial to x final of $mg \sin \theta$ times dx .

And that's precisely our second piece here.

So here's the simple application of the work energy theorem.